Acta Mechanica Solida Sinica, Vol. 29, No. 3, June, 2016 ISSN 0894-9166
Published by AMSS Press, Wuhan, China

Analytical Solutions for Free Vibration and Buckling of Composite Beams Using a Higher Order Beam Theory**

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Received 19 Novermber 2014, revision received 8 March 2016

ABSTRACT To satisfy the interfacial shear force continuity conditions, a new model is proposed for the two-layer composite beam with partial interaction by modifying Reddy's higher order beam theory. The governing differential equations for free vibration and buckling are formulated using the Hamilton's principle, the natural frequencies and axial forces are thus analytically obtained by Laplace transform technique. The analytical results are verified through the comparison with those of several other models common in use; and the presented model is found to be a finer one than the Reddy's. A parametric study is also performed to investigate the effects of geometry and material parameters.

KEY WORDS Reddy's higher order beam theory, Timoshenko beam theory, composite beams, free vibration and buckling, Laplace transform

I. Introduction

Composite beams with interlayer slip are becoming increasingly widespread in many engineering fields, because of appropriate usage of the mechanical properties of each component, e.g. the high tensile strength of steel and the high compressive strength of concrete in the steel-concrete composite beams have been effectively utilized. Usually, the sub-layers of composite beams are connected by flexible shear connectors such as headed studs, interlayer slip may thus occur even subject to small loads. Hence, in the early days Newmark et al.[1] proposed a model for the two-layer composite beam with partial interaction based on the Euler-Bernoulli beam theory (EBT), where the shear strain of each sub-layer was neglected and the displacement was assumed small. To capture more fidelity of the structure and the shear effects which the EBT can't cope with, more refined composite beam kinematics were proposed subsequently, e.g. studies[2–8] focused on the first order shear deformation of composite beam components. Based on the first order shear deformation theory (FSDT), various analyses have been carried out on the mechanical behaviors of composite beams, including static response[9–13], free vibration[7,8] and buckling[2,3,8]. However, what has to be noted is that the shear correction factor introduced by the Timoshenko beam theory (TBT) is attributed to the geometry of cross-section of each sub-layer as well as the shear stress on the cross-section[14], i.e. the factor is no longer constant during the deformation of composite beams. To overcome this drawback, it is a good choice to use higher order shear deformation theory (HSDT) for its free requirement on the shear correction factor.

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** Project supported by the National High Technology Research and Development Program of China (No. 2009AA032303-2).

Therefore, the HSDT has received much attention[14–16] in formulating the sub-layers, by making use of its elaborate kinematics. And it is much finer than the FSDT-based kinematcis. For example, the Reddy's HSDT[17] assumes that the axial displacement of cross-section varies over the beam depth as a cubic polynomial and a quadratic distribution of shear stress along the beam depth can be captured. Utilizing this type of HSDT[17], Chakrabarti et al.[15,16] studied the static response of two-layer composite beams within linear elastic range; and Chakrabarti et al.[14] extended it to the range of dynamics problems using the finite element method (FEM). He and Yang[18,19] modified Reddy's HSDT[14–16] to ensure the satisfaction of interfacial shear force continuity condition of two-layer composite beams, and the static and buckling analyses were carried out using the exact method[19] and the FEM[18], respectively. In addition, based on two types of HSDTs, Subramanian[20] developed a displacement-based finite element and an analytical procedure for the free vibration analysis of composite laminated beams; and Li et al.[21] developed an exact finite element to conduct the free vibration analysis of composite laminated beams using the hyperbolic shear deformation theory.

Despite the convenience and wide applicability of the FEM in analyzing composite structures, the exact closed form solutions are still required for the analysis as a benchmark. Therefore, analytical solutions for composite beams have been proposed. Based on the classical Newmark model[1], a large number of researchers[22–24] analytically studied the free vibration characteristics of two-layer composite beams within linear elastic range; many scholars[24–26] also analytically explored the stability characteristics of two-layer composite columns. To evaluate the shear effects, plenty of investigators[2,3,5,7,8,27] obtained the exact analytical solutions for static, free vibration and buckling problems within linear elastic range. However, to the best knowledge of the authors, studies on the exact free vibration and buckling analyses of two-layer composite beams using the HSDT are quite few, due to the increasing complexity in the mathematical models introduced by the HSDT.

The main objective of this study is to improve the aforementioned studies of Refs.[18,19]. In this study, the modified Reddy's HSDT[18,19] is used in the modeling, and the dynamic partial differential equations governing the higher order shear deformation of two-layer composite beams are formulated by the Hamilton's principle. A novel exact analytical method based on Laplace transform is initially developed to solve the boundary value problems of complex ordinary differential equations (ODEs) for free vibration and buckling analyses. Compared with the FEM, the proposed analytical method is free from convergence problem, which is inevitable for the FEM due to mesh-dependence. In addition, unlike the Newton-like iteration method, the bisection method is part of the proposed one, and is thus stable for the solution of transcendental equations for eigenvalues. With the help of the proposed method, the troublesome procedure of decoupling in conventional solving techniques is avoided; and it is potentially capable of solving other mathematical problems governed by the ODE systems. The exact results are verified through the comparison with the results of models based on the two-dimensional theory, which are TBT and Reddy's HSDT. Four combinations of boundary conditions common in engineering are considered in the numerical verification, and the reliability of the presented mathematical model and the exact solving method are demonstrated by the excellent agreement of performances with the two-dimensional model. Moreover, parametric studies on the slenderness ratio of composite beams, stiffness of shear connectors and different boundary conditions are conducted to examine the shear effects on the dynamic and stability characteristics.

II. Formulations

2.1. Description of problems and assumptions

A straight, planar, two-layer composite beam is being considered with different cross-sections, materials, and flexible shear connectors uniformly smeared over the interface. Sub-layers of the entire span L, as shown in Fig.1(a), are marked with c and s, respectively. The layers c and s are respectively placed in the Cartesian coordinate systems oxy_{c} and oxy_{s} originated from the centroid of each layer at the left end of the span. These two centroid axes and the shear interface divide the overall depth of the composite beams into four parts, i.e. h_1, h_2, h_3 and h_4. The axial displacement assumption for each layer is shown in Fig.1(b), where u_{c0} and u_{s0} indicate the axial displacements at the centroids of cross-sections c and s, respectively; and u_{cs} indicates the interfacial slip between the two layers. Parameters θ_{c} and θ_{s} represent the tangential slopes at the centroids of cross-sections c and s, respectively.