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Full state hybrid projective synchronization in continuous-time chaotic (hyperchaotic) systems ☆

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Abstract

Chaos synchronization, as an important topic, has become an active research subject in nonlinear science. Over the past two decades, chaos synchronization between nonlinear systems has been extensively studied, and many types of synchronization have been announced. This paper introduces another novel type of chaos synchronization – full state hybrid projective synchronization (FSHPS), which includes complete synchronization, anti-synchronization and projective synchronization as its special item. Based on the Lyapunov's direct method, the general FSHPS scheme is given and illustrated with Lorenz chaotic system and hyperchaotic Chen system as examples. Numerical simulations are used to verify the effectiveness of the proposed scheme.

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Keywords: FSHPS; Lyapunov's direct method; Lorenz system; Hyperchaotic Chen system

1. Introduction

Chaos plays a more and more important role in nonlinear science field. Chaos synchronization has received a significant attention in the last few years due to its potential applications [1–19]. Since Pecora and Carroll [2] presented the chaos synchronization method to synchronize two identical chaotic systems with different initial conditions in 1990, many different methods have been reported to investigate chaos synchronization of some types of chaotic (hyperchaotic) attractors. Among them the following methods are in common use: linear feedback control method [3,4], nonlinear feedback control method [5–8], adaptive control method [9], observer-based control method [11,14].

There exist many types of synchronization such as complete synchronization [2], anti-synchronization [9]. Mainieri and Rehacek [10] reported a new form of chaos synchronization, termed as projective synchroniza-

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tion, that the drive and response systems could be synchronized up to a scaling factor α (a proportional relation), which is usually observable in a class of systems with partial linearity, such as Lorenz system. The special cases of projective synchronization where $\alpha = 1$ and $\alpha = -1$ are complete synchronization and anti-synchronization, respectively. Recently, Wen [11] presented a full-state projective synchronization between two dynamical systems. For two dynamical systems

$$\dot{x}(t) = F(x) \leftarrow \text{drive system,} \tag{a}$$

$$\dot{y}(t) = G(x,y) \leftarrow \text{response system,} \tag{b}$$

where $x = (x_1, x_2, \ldots, x_n)^{\mathrm{T}}$, $y = (y_1, y_2, \ldots, y_n)^{\mathrm{T}} \in R^n$ are state variables of the drive system (a) and the response system (b), respectively. If there exists a nonzero constant α such that $\lim_{t\to\infty}\|y - \alpha x\| = 0$ i.e., $\lim_{t\to\infty}|y_i - \alpha x_i| = 0$, $i = 1, 2, \ldots, n$, then we call them full-state projective synchronization. Projective synchronization attracted lots of attention to study [10–18] because of its proportionality between the synchronized dynamical states. In application to secure communications [17–19], this feature can be used to extend binary digital to M-nary digital communication [18] for achieving fast communication. Inspired by above works, in this paper, we will investigate the following synchronization phenomenon:

Definition 1 (*FSHPS*). For the drive system (a) and response system (b), it is said that the drive system (a) and response system (b) are full state hybrid projection synchronization (FSHPS), if there exists a nonzero constant matrix $\alpha = \mathrm{diag}(\alpha_1, \alpha_2, \ldots, \alpha_n) \in R^{n\times n}$ such that $\lim_{t\to\infty}\|y - \alpha x\| = 0$ i.e., $\lim_{t\to\infty}|y_i - \alpha_i x_i| = 0$, $i = 1, 2, \ldots, n$.

The paper is organized as follows. In Section 2, the FSHPS scheme is presented briefly. In Section 3, the scheme is applied to investigate FSHPS between two identical Lorenz chaotic system and two identical hyperchaotic Chen system, respectively, and numerical simulations are used to verify the effectiveness of the proposed scheme. Finally, a conclusion ends the paper.

2. General FSHPS scheme

Consider a class of n-dimensional chaotic system in the form of

$$\dot{x} = F(x), \tag{1}$$

where $x = (x_1, x_2, \ldots, x_n)^{\mathrm{T}} \in R^n$ is the state vector, and $F(x) = (F_1(x), F_2(x), \ldots, F_n(x))^{\mathrm{T}} \in R^n$ is continuous nonlinear vector function.

We refer to (1) as the drive system and the response system is given by

$$\dot{y} = F(y) + u, \tag{2}$$

where $y = (y_1, y_2, \ldots, y_n)^{\mathrm{T}} \in R^n$, $F(y) = (F_1(y), F_2(y), \cdots, F_n(y))^{\mathrm{T}} \in R^n$. $u = u(x,y) = (u_1(x,y), u_2(x,y), \ldots, u_n(x,y))^{\mathrm{T}} \in R^n$ is the controller to be determined for the purpose of full state hybrid projective synchronization. Let the vector error state be $e(t) = y(t) - \alpha x(t)$, where α is a n-order diagonal matrix, i.e., $\alpha = \mathrm{diag}(\alpha_1, \alpha_2, \ldots, \alpha_n)$, $\alpha_i \in R$, $i = 1, 2, \ldots, n$. Thus the error dynamical system between the drive system (1) and the response system (2) is

$$\dot{e}(t) = \dot{y} - \alpha\dot{x} = \widetilde{F}(x,y) + u, \tag{3}$$

where $\widetilde{F}(x,y) = F(y) - \alpha F(x) = (F_1(y) - \alpha_1 F_1(x), F_2(y) - \alpha_2 F_2(x), \ldots, F_n(y) - \alpha_n F_n(x))^{\mathrm{T}}$.

In the following, we will give a principle to find suitable feedback controller u such that the two chaotic (hyperchaotic) systems are FSHPS. Construct a dynamical Lyapunov function

$$V = \frac{1}{2}\mathrm{e}^{\mathrm{T}}Pe, \tag{4}$$

where P is a positive definite constant matrix. One may choose P as the corresponding identity matrix in most case. The time derivative of V along the trajectories of Eq. (3) is

$$\dot{V} = \mathrm{e}^{\mathrm{T}}P(u + \widetilde{F}). \tag{5}$$