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Analytical study of the nonlinear dust-acoustic waves in an unmagnetized dusty plasma

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Abstract

The nonlinear dust-acoustic waves in an unmagnetized dusty plasma, including consideration of the dust charge variation, is analytically investigated by using the formally variable separation approach. The exact analytical solutions in the general case are also obtained.

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Keywords: The nonlinear dust-acoustic waves; Plasmas; The formally variable separation approach; Exact analytical solution

1. Introduction

Nowadays, there is a growing interest in the study of different types of collective processes in dusty plasma and other nonlinear physics [1–5]. It has been shown that the presence of extremely massive and highly charged dust grains modifies the existing plasma wave spectra [2]. Motivated by some theoretical and experimental studies [3–6], some authors have studied the dust acoustic solitary structure in a dusty plasma model consisting of negatively charged dust fluid and isothermal or non-isothermal ions, and also non-thermal dust [7,8]. Recently, Ma and Liu [9], Xie et al. [10,11,13] and Chen and Liu [12] have considered the effects for the dust charge variation and have shown the existence of dust acoustic solitons. Numerical simulation studies [14] on linear and nonlinear dust acoustic waves exhibit a significant amount of ion trapping in the wave potential. Clearly, there is a departure from the Boltzmann ion distribution and one encounters vortex-like ion distribution in phase space. Tsytovich and Angelis [15–19] have fully described the kinetic theory of dusty plasmas by the numbers and progressed many important theory research for the hydrodynamic equations in dusty plasmas. Duan et al. [20–22] have investigated the nonlinear dust acoustic waves in dusty plasmas with many different dust grain, and have shown that the nonlinear dust acoustic wave can be described by the modified Korteweg–de Vries equation, the modified Kadomtsev–Petviashvili equation and the Zakharov–Kuznetsov

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equation, respectively. Lin et al. [23–26] have analytically investigated the nonlinear waves in nonlinear physics and the nonlinear Debye screening in plasmas by using the formally variable separation approach [27–29]. These theoretical results have established scientific values and application prospects for the fundamental research in dusty plasmas.

In this letter, we reconsider a one-dimensional propagation of dust-acoustic waves in a unmagnetized dusty plasma from the another mathematical angle, avoiding the reductive perturbation technique. The nonlinear dust-acoustic waves in an unmagnetized dusty plasma, including consideration of the dust charge variation, is analytically investigated by using the formally variable separation approach. The exact analytical solutions in the general case are also obtained. It is found that the equations of the motion for this system due to the formally variable separation approach causes the nonlinear dust acoustic waves to be accurately described, and new kinds of the analytical solutions for the nonlinear dust acoustic waves are produced.

2. Mathematical formalism

We consider an unmagnetized plasma consisting of ions, electrons, and cold, extremely massive, microsized, negatively charged dust fluid. The basic equations governing the dusty plasmas are [10]

$$\frac{\partial n_{\mathrm{d}}}{\partial t}+\frac{\partial}{\partial x}(n_{\mathrm{d}}u_{\mathrm{d}})=0, \tag{2.1}$$

$$\frac{\partial u_{\mathrm{d}}}{\partial t}+u_{\mathrm{d}}\frac{\partial u_{\mathrm{d}}}{\partial x}=Z_{\mathrm{d}}\frac{\partial \phi}{\partial x}, \tag{2.2}$$

$$\frac{\partial^2 \phi}{\partial x^2}=Z_{\mathrm{d}}n_{\mathrm{d}}+n_{\mathrm{e}}-n_{\mathrm{i}}, \tag{2.3}$$

where we have normalized the quantities t, x, n_{d}, u_{d} and ϕ by n_{d0}, $\omega_{\mathrm{d}}^{-1}=(m_{\mathrm{d}}/4\pi n_{\mathrm{d0}}Z_{\mathrm{d0}}^2e^2)^{1/2}$, $\lambda_{Dd}=(T_{\mathrm{eff}}/4\pi Z_{\mathrm{d0}}n_{\mathrm{d0}}e^2)^{1/2}$, $C_{\mathrm{d}}=(Z_{\mathrm{d0}}T_{\mathrm{eff}}/m_{\mathrm{d}})^{1/2}$, and T_{eff}/e, respectively; Z_{d0} is the unperturbed number of charges residing on the dust particles measured in units of the electron charge, and $T_{\mathrm{eff}}=T_{\mathrm{i}}T_{\mathrm{e}}/(\mu T_{\mathrm{e}}+\nu T_{\mathrm{i}})$.

Here, the ion number density and electron number density satisfy the Boltzmann distribution. Because the motion of dust is not so fast that the contribution from the electron current to the dust is balanced by the ions. By using the intricate algebra for calculates of the charge current balance equation [10,30], Xue have obtained the normalized dust charge [31]

$$\begin{aligned} n_{\mathrm{i}}&=\mu\exp(-s\phi),\\ n_{\mathrm{e}}&=\nu\exp(\beta s\phi),\\ Z_{\mathrm{d}}&=1+\gamma_1\phi+\gamma_2\phi^2+\cdots, \end{aligned} \tag{2.4}$$

where $\beta=T_{\mathrm{i}}/T_{\mathrm{e}}$ is the ratio of ion temperature to electron temperature and $\delta=n_{\mathrm{i0}}/n_{\mathrm{e0}}$ is the ratio of the number density of ions to the number density of electrons, $s=1/(\mu+\nu\beta)$, $\mu=\delta/(\delta-1)$, $\nu=1/(\delta-1)$, and the other notation has its usual meaning. The variables γ_j which are related to the physical parameters, describe the effects of dust charge variation.

We extend the formally variable separation approach [27–29] to the governing equations for this system. The formally variable separation equations are

$$\varphi_t=K_1(\varphi),\quad \varphi_x=K_2(\varphi), \tag{2.5}$$

where $\varphi=\varphi(t,x)$ is a scalar function. The sole possible solution of the compatible nature condition $\varphi_{xt}-\varphi_{tx}=0$ is

$$K_1=\alpha_1K(\varphi),\quad K_2=\alpha_2K(\varphi), \tag{2.6}$$

where α_1 and α_2 are constants. We consider the nonlinear dust acoustic waves as some analytical functions, which contacts with the formally variable as the physical quantities depending on φ,

$$\phi(t,x)=\Phi(\varphi),\quad n_{\mathrm{d}}(t,x)=N_{\mathrm{d}}(\varphi),\quad u_{\mathrm{d}}(t,x)=U_{\mathrm{d}}(\varphi). \tag{2.7}$$

Thus, the basic equations governing the dusty plasmas can be transformed as