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Communications in Nonlinear Science and Numerical Simulation 12 (2007) 1291–1301
Communications in Nonlinear Science and Numerical Simulation
www.elsevier.com/locate/cnsns

Perturbation analysis of entrainment in a micromechanical limit cycle oscillator

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Received 11 November 2005; received in revised form 24 January 2006; accepted 24 January 2006
Available online 20 March 2006

Abstract

We study the dynamics of a thermo-mechanical model for a forced disc shaped, micromechanical limit cycle oscillator. The forcing can be accomplished either parametrically, by modulating the laser beam incident on the oscillator, or non-parametrically, using inertial driving. The system exhibits both 2:1 and 1:1 resonances, as well as quasiperiodic motions and hysteresis. A perturbation method is used to derive slow flow equations, which are then studied using the software packages AUTO and pplane7. Results show that the model agrees well with experiments. Details of the slow flow behavior explain how and where transitions into and out of entrainment occur.

PACS: 02.30.Hq; 02.30.Oz; 46.40.Ff

Keywords: Resonance; MEMS; Bifurcation; Perturbation

1. Introduction

Entrainment is the phenomenon in which freely oscillating systems, synchronize with each other or with an external force. Entrainment can occur in numerous physical, chemical, biological and sociological systems. Examples of such systems include entrainment of human circadian rhythms by light, where the biological clock is entrained to the cycle of day and night [1], radio frequency systems [2], superconducting Josephson junction arrays [3] and the mutual entrainment of fireflies [4] which glow in unison after synchronization.

In this paper, we use perturbation methods to analyze the entrainment behavior exhibited by a planar, disc-shaped micromechanical limit cycle oscillator. The oscillator, shown in Fig. 1, consists of a thin circular plate of single crystal Si supported above a Si substrate by a SiO_2 pillar [5,6]. A constant (CW) laser beam, focused to a 5 μm spot near the edge of the disc, is used both to detect the vibrations and to drive the disc. The disc is thin enough that much of the incident laser light is transmitted through the disc and then reflected back by the Si substrate below. This process repeats itself in a series of reflections and transmissions, the net result of which

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1007-5704/$ - see front matter
doi:10.1016/j.cnsns.2006.01.017

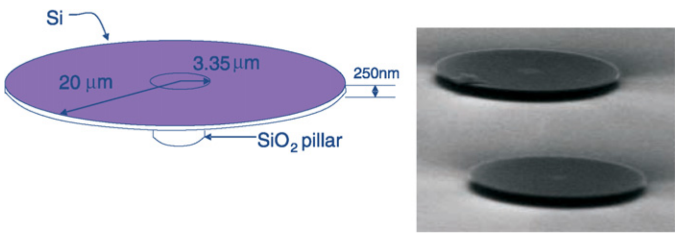

Fig. 1. Disk-shaped oscillator. Right: SEM image of actual structure. From [5].

is that the disc-substrate system forms a Fabry–Perot interferometer. Both the net reflected and the net absorbed light vary periodically with the deflection of the disc at the point of illumination. Thus the laser can be used to interferometrically detect vibrations of the disc. In addition, the amount of heat absorbed and hence the thermal strains also vary with disc deflection. The oscillatory thermal strains produce a thermal drive of the disc and they modulate the disc's stiffness [7]. Since the thermal driving force depends on the position of the disc, the system has feedback that can cause the rest position of the disc to become unstable when the laser power exceeds a threshold, driving the disc into limit cycle motions via a Hopf bifurcation [7].

In the experiments [5] the CW laser power was increased to a level just beyond the threshold for limit cycle oscillations. Once the disc was oscillating, it was found that the frequency of vibration can be tuned by applying a "pilot signal" consisting of either a modulation of the incident laser beam or an inertial drive provided by a modulated piezoelectric actuator taped to the back of the chip containing the disc. If the frequency of modulation of the pilot is close to the limit cycle frequency of the oscillator, the oscillator locks itself onto the pilot signal and remains locked in frequency and phase over a range of frequencies. The disc is said to have been entrained by the pilot signal. If the pilot frequency is not close to the oscillator limit cycle frequency, then the oscillator continues to oscillate at its own frequency and phase. The system exhibits hysteresis, that is, the entrainment region obtained when sweeping backward in frequency has different boundaries than the comparable region obtained when sweeping forward. The amplitude and the frequency response for the case of inertial (piezo) pilot signal are shown in Figs. 2 and 3. See [5] for further details of the experiments.

Modeling of this device and numerical simulations of the governing equations have been discussed earlier in [7–9]. Here we use perturbation theory to discern additional details of the transitions into and out of entrainment that are not amenable to numerical simulations. The model equations for this system are [7,9]:

$$\ddot{z} + \frac{1}{Q}(\dot{z} - D\dot{T}) + (1 + CT)(z - DT + \beta(z - DT)^3) = M\sin(\omega_{\text{piezo}}t), \tag{1}$$

$$\dot{T} + BT = AP, \tag{2}$$

and

$$P = P_{\text{laser}}(1 + \varphi\cos(\omega_{\text{laser}}t))(\alpha + \gamma\sin^2(2\pi(z - z_0))). \tag{3}$$

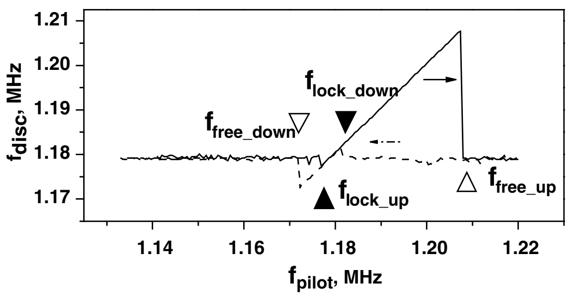

Fig. 2. Frequency of response versus piezo forcing frequency, obtained experimentally. From [5].