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Nonlinear processes in Hamiltonian reconnection

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ABSTRACT

The generation of small spatial scales and their interplay with large scale coherent structures is one of the outstanding phenomena of plasma physics and fluid mechanics. In high temperature space and laboratory plasmas dissipative effects become important at length scales that are much smaller than those where microscopic dynamical effects, related e.g., to electron inertia, come into play. Here we discuss the role of this dissipationless small scale dynamics on the nonlinear evolution of collisionless magnetic reconnection within the framework of the so called "two-field" and "four-field models".

1. Introduction

Plasmas in most configurations of interest in astrophysics, space physics and in laboratory magnetic fusion experiments are essentially dissipationless systems where relaxation processes due to binary collisions are weak and their effect is negligible even for the relatively slow magnetohydrodynamic (MHD) processes. In the ideal MHD description, where the plasma is thought to behave as a perfect conductor, plasma elements initially connected by a magnetic field line remain connected as they move. These "magnetic connections" constrain the plasma dynamics by forbidding transitions between configurations with the same total energy but different magnetic topology. Violations of this property lead to so-called "magnetic reconnection" (MR) [1,2] processes. Even though these violations occur in narrow spatial regions, they have important consequences as they allow a global rearrangement of the magnetic connections. Reconnection can be described pictorially as a local cutting and re-knitting of field lines and is driven by the strong local current inhomogeneities that the nonlinear plasma dynamics creates because of the magnetic topology constraints. This process is accompanied by a significant transformation of magnetic energy into plasma energy. Solar flares and, in the laboratory, the disruptions of the plasma confinement related to the so-called saw-tooth oscillations are well known examples of such energy transformations.

MR can occur in the presence of large localized currents limited by a small electrical resistivity caused by Coulomb collisions. However, in the high temperature rarefied plasmas, encountered in astrophysics or in present day fusion experiments, Coulomb collisions are very infrequent and are an inefficient mechanism for breaking field lines. In such regimes plasma currents are mainly limited by non-dissipative effects, such as the inertia of the electrons or wave particle

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resonances, which can break magnetic connections but, contrary to resistivity, maintain the Hamiltonian character of the plasma dynamics. An outstanding question is therefore whether reconnection and dissipation need to be linked and, if not, how Hamiltonian reconnection evolves in the absence of dissipation.

In this article we present new results concerning the nonlinear dynamics of Hamiltonian MR, investigated by means of a four-field (4F) model, and compare these with previous results obtained from a simpler two-field (2F) model. The comparison is carried out from both an analytical and a numerical point of view, by making use of simulations carried out in the large Δ' regime, with Δ' indicating the standard instability parameter for reconnection instabilities [3].

2. Mixing of the Lagrangian invariants and current and vorticity layers

The first answer to the above questions was presented for two dimensional magnetic configurations in Ref. [4]. In this reference it was shown that even in the absence of dissipation and in the related presence of an infinite number of conserved quantities [5], the so-called Casimir invariants (see e.g. [6]), MR can proceed unimpeded and can lead to a transition between two macroscopic configurations. This is made possible by the fact that the excess energy is transported towards increasingly small spatial scalelengths and that the topological constraints arising from the Casimir conservation are loosened by a mixing process that involves again the formation of increasingly small spatial scalelengths. At the same time it was stressed that the topological constraints are actually responsible for the formation of the current and vorticity layers, determine their spatial structure and thus control the nonlinear evolution of the reconnection process.

2.1. Two-field model, a summary

In Ref.[4] a simplified plasma description was adopted where the plasma dynamics is reduced to the dynamics of two two-dimensional scalar fields. Here we do not provide an explicit derivation of the equations that govern the time evolution of these fields as it can be found in Ref. [7]. The aim here is to exemplify the formation of small spatial scales, the role of the Lagrangian invariants and their mixing process on a simpler set of equations having in mind that similar considerations will apply to the more extended 4F model.

The simplest description of the reconnection processes due to the effect of electron inertia in a two dimensional plasma configuration is given in terms of two equations that account for the combined fluid electron and ion dynamics in the x–y plane. In formulating these equations it is assumed that the plasma is embedded in a strong inhomogeneous magnetic field of the form

$$\mathbf{B} = B_0\mathbf{e}_z + \nabla\psi \times \mathbf{e}_z, \tag{1}$$

where B_0 is constant and $\psi(x, y, t)$ is the magnetic flux function. Furthermore it is assumed that the plasma flow is incompressible and that it is given in terms of the stream function $\varphi(x, y, t)$ by

$$\mathbf{v} = \mathbf{e}_z \times \nabla\varphi. \tag{2}$$

The functions ψ and φ represent the two fields of the model and their evolution equations can be written in a Lie–Poisson form [5]

$$\frac{\partial(\psi - d_e^2\nabla^2\psi)}{\partial t} + [\varphi, \psi - d_e^2\nabla^2\psi] + \rho_s^2[\psi, \nabla^2\varphi] = 0, \tag{3}$$

$$\frac{\partial\nabla^2\varphi}{\partial t} + [\varphi, \nabla^2\varphi] - [\psi, \nabla^2\psi] = 0, \tag{4}$$

where d_e and ϱ_s are microscopic scale lengths related to electron inertia and to parallel electron compressibility respectively and the Poisson brackets are defined as $[A, B] = \mathbf{e}_z \cdot \nabla A \times \nabla B$.

The structural properties of the system evolution become more evident if we express ψ and φ in terms of the two Lagrangian invariant fields, $G_\pm$,

$$G_\pm = \psi - d_e^2\nabla^2\psi \pm d_e\varrho_s\nabla^2\varphi. \tag{5}$$

In these new fields the evolution equations take the form

$$\frac{\partial G_\pm}{\partial t} + [\varphi_\pm, G_\pm] = 0, \tag{6}$$

where

$$\varphi_\pm = \varphi \pm (\varrho_s/d_e)\psi. \tag{7}$$

The Lagrangian invariants $G_\pm$ are advected by the generalized velocity fields $\mathbf{v}_\pm = \mathbf{e}_z \times \nabla\varphi_\pm$. This implies that the system evolution is constrained by the two infinite families of Casimir invariants given by the functionals of the form $\int dxdy\mathscr{F}(G_\pm)$. The system of Eqs. (6) is closed by a Yukawa-like equation for the magnetic flux function