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Dynamics of particle trajectories in a Rayleigh–Bénard problem

C. Simó [a], D. Puigjaner [b,*], J. Herrero [c], F. Giralt [c]

[a] Dept. de Matemàtica Aplicada i Anàlisi, Univ. de Barcelona, Barcelona, Catalunya, Spain
[b] Dept. Enginyeria Informàtica i Matemàtiques, Univ. Rovira i Virgili, Tarragona, Catalunya, Spain
[c] Dept. Enginyeria Quimica, Univ. Rovira i Virgili, Tarragona, Catalunya, Spain

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Fluid particle trajectories for the Rayleigh–Bénard problem in a cube with perfectly conducting lateral walls have been investigated. The velocity and temperature fields of the stationary flow solutions have been obtained by means of a parameter continuation procedure based on a Galerkin spectral method. The rich dynamics of the resulting fluid particle paths has been studied for three branches of stationary solutions and different values of the Rayleigh number within the range $10^4 \leqslant Ra \leqslant 1.5 \times 10^5$ at a Prandtl number equal to 130. The stability properties and bifurcations of fixed points, which play a key role in the global dynamics, have been analyzed. Main periodic orbits and their stability character have also been determined. Poincaré maps reveal that regions of chaotic motion and regions of regular motion coexist inside the cavity. The boundaries of these three-dimensional regions have been determined. The metric entropy gives an indication of the mixing properties of the large chaotic zone.

1. Introduction

The determination and control of fluid mixing efficiency is an important issue in many engineering applications. Although efficient mixing is usually related to turbulent regimes there are several industrial applications that require an efficient mixing in the absence of turbulence or high shear stresses. In such cases, concepts and tools from dynamical systems theory can be applied to understand the mixing process. While the pioneering work of Reynolds connected fluid mixing and chaos theory [1] it was not until the seminal paper of Aref [2] that a mixing fluid flow problem was thoroughly studied from a dynamical systems perspective. Aref [2] introduced the term chaotic advection to refer to the complex behavior that a passive fluid particle, or a passively advected scalar quantity such as temperature or concentration can attain as it moves with the flow. Chaotic advection has received significant attention in recent years [3–11] because of its impact on both transport and mixing properties in many fluid flow systems.

Passive fluid advection is described by the Lagrangian representation of fluid motion. Even when the fluid flow is laminar and steady and the Eulerian velocity at any given point in space is fixed, the trajectories of passive particles may exhibit rich chaotic dynamics (i.e., trajectories of two arbitrarily close points in space may separate exponentially in time). Mixing induced by molecular diffusion in chaotic regions of the flow, is often negligible compared to mixing induced by the flow dynamics. It is known that mixing is enhanced when the levels of chaos in the flow domain increase [7]. On the other hand, the existence of regions of regular motion, characterized by the presence of islands in two-dimensional (2D) maps and nested KAM-tori in three-dimensional (3D) flows, form a barrier that prevents the fluid inside the regular region from mixing

* Corresponding author. Tel.: +34 977559663; fax: +34 977559710.
E-mail addresses: carles@maia.ub.es (C. Simó), dolors.puigjaner@urv.cat (D. Puigjaner), joan.herrero@urv.cat (J. Herrero), fgiralt@urv.cat (F. Giralt).

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with the outside fluid. Hence, the global geometrical viewpoint of dynamical systems theory can be very useful in the understanding of qualitative and quantitative aspects of fluid motion.

The main objective of this work is to present some preliminary results on the application of dynamical systems tools to the analysis of the rich dynamics of particle trajectories in a particular 3D, volume-preserving, fluid flow system. The fluid flow problem investigated is the natural convection (Rayleigh–Bénard) inside a cubical cavity heated from below, with perfectly conducting lateral walls. The bifurcation diagram of steady convective flow patterns in this flow system, recently reported by Puigjaner et al. [12], reveals the existence of four and nine different stable complex flow patterns in the range $Ra < 1.5 \times 10^5$ at $Pr = 0.71$ and 130, respectively. The complexity of the bifurcation diagrams is especially noticeable at $Pr = 130$, where all solutions branches that set in at bifurcations from the conductive solution, except for the single roll denoted as B_1, are connected to each other through their subsequent bifurcations [12].

This study focuses on the analysis of three particular steady flow patterns, denoted as B_2, B_{25} and B_{251}, at $Pr = 130$. The choice of these three flow patterns was motivated by their connection in the bifurcation diagram through symmetry-breaking bifurcations and by the complex evolution of the spatial configuration of the B_2 flow pattern as the Rayleigh parameter increased. Moreover, all three flow patterns are stable within certain ranges of Ra in the studied domain. To explore the dynamics of these three flow patterns several tools, such as Poincaré sections, critical-point analysis, periodic orbits and Lyapunov exponents have been used. Some topological relations have also been considered in order to check the results obtained. The symmetries of the solutions have been exploited to obtain invariant closed surfaces where the velocity field is everywhere tangent to the surface. Invariant surfaces are particularly important because they separate the domain into non-communicating regions, that is, into regions between which there is no transport. In the current work the dynamics of the flows restricted over invariant surfaces have been analyzed with the objective to obtain useful knowledge about the global flow dynamics. In addition, the effect of the Rayleigh number on the number and type of critical points and on the formation and persistence of regions with regular motion has been investigated. The metric entropy has been used as a quantitative measure of the mixing properties of the flow.

2. Problem formulation and numerical methods

2.1. Flow system

An incompressible steady flow of a Newtonian fluid confined in a cube with six rigid walls is studied. The top and bottom horizontal walls are kept at constant temperatures T_c and T_h ($T_c < T_h$), respectively, and the four lateral walls are assumed to be perfectly conducting, i.e., a linear vertical temperature profile is assumed. The domain, shown in Fig. 1, is scaled by the length of the side of the cube L and is represented by $\Omega = [-1/2, 1/2] \times [-1/2, 1/2] \times [-1/2, 1/2]$. The problem is governed by the 3D incompressible Navier–Stokes equations and the energy conservation equation. The flow within the cavity depends on two non-dimensional parameters, the Rayleigh number, Ra, and the Prandtl number, Pr. These parameters are defined as $Ra = \beta(\Delta T)gL^3/\alpha\nu$ and $Pr = \nu/\alpha$, where g is the acceleration of gravity, β is the coefficient of thermal expansion, ν is the kinematic viscosity, α is the thermal diffusivity and $\Delta T = T_h - T_c$. The Prandtl number ($Pr = 130$) is chosen to match silicone oil.

2.2. Velocity field and bifurcation diagram

The numerical method followed elsewhere [12,13] to obtain steady solutions of the Boussinesq approximation of the governing equations is briefly described here. Calculations were based on a Galerkin spectral method with a complete, divergence-free set of basis functions. Once the velocity and temperature fields were approximated by truncated expansions in terms of the basis functions, the partial differential governing equations were transformed into a system of ordinary differential equations whose unknowns were the coefficients in the expansions. A continuation procedure was applied to determine steady solutions at different Rayleigh numbers. The bifurcations and stability character of the solutions were determined along the different tracked solution branches. A total set of 11,076 basis functions were typically used in the calculations. The symmetry properties of the solutions along a bifurcation branch, when they exist, were exploited to reduce the

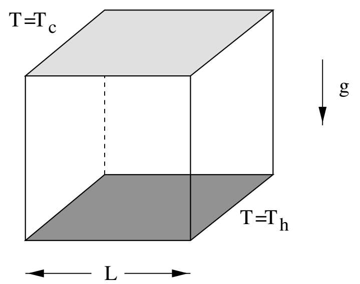

Fig. 1. A sketch of the cubical cavity.