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On synchronization of a forced delay dynamical system via the Galerkin approximation

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Abstract

A forced scalar delay dynamical system is analyzed from the perspective of bifurcation and synchronization. In general first order differential equations do not exhibit chaos, but introduction of a delay feedback makes the system infinite dimensional and shows chaoticity. In order to study the dynamics of such a system, Galerkin projection technique is used to obtain a finite dimensional set of ordinary differential equations from the delay differential equation. We compare the results of simulation with those obtained from direct numerical simulation of the delay equation to ascertain the accuracy of the truncation process in the Galerkin approximation. We have considered two cases, one with five and the other with eight shape functions. Next we study two types of synchronization by considering coupling of the above derived equations with a forced dynamical system without delay. Our analysis shows that it is possible to have synchronization between two such systems. It has been shown that the chaotic system with delay feedback can drive the system without delay to achieve synchronization and the opposite case is also equally valid. This is confirmed by the evaluation of the conditional Lyapunov exponents of the systems.

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1. Introduction

Delay induced instabilities as described by delay differential equations, play an important role in modelling natural phenomena. Such models are used in many different scientific disciplines like electronics, laser physics, ecology, engineering, economics and cognitive sciences [1–5]. As the dynamical systems given by DDE's have an infinite dimensional state space, the attractors of the solutions are also high dimensional. The numerical simulation method for solving such system of equations is mainly based on interpolation approximations

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giving rise to continuous version of Runge–Kutta method. Recently, a new method was suggested by Györi et al. [6] which is based on the approximation of the functional differential equations by equations with piecewise constant arguments. So the projection of DDE's into ODE's came up as an alternative method of solving them and this simplification on the other hand makes it easier to perform other numerical studies applicable to a wide variety of applied problems. The method used here for such a transformation is the Galerkin approximation technique [7–9]. It is observed that the dynamics of the DDE's can be obtained very close to the values obtained by direct numerical simulation if a moderate number of shape functions are taken into account.

One of the fundamental task to classify the behaviour of a system is to estimate the Lyapunov exponents. They account for exponential convergence or divergence of trajectories that start close to each other [10]. For practical applications, it is important to know the largest Lyapunov exponent, because a positive largest Lyapunov indicates the system of being chaotic. The algorithms for calculating Lyapunov exponents for continuous and differentiable ordinary differential equation have been developed [11,12] and later improved and modified by various scientists [13,14]. such algorithms, however, do not work for the system with discontinuities or time delay [15]. An alternative approach was suggested by Farmer [16] long back. So in such a case the estimation of the Lyapunov exponent is not straightforward. So when DDE's are transformed into ODE's, one can again follow the usual algorithms for the estimation.

In this context it is worth while to mention that a very innovative and new approach to time delayed system has been adopted by Steve Suh et al. [17], who employed the Hilbert–Huang transformation procedure to unfold the chaotic nature of a nonlinear delay differential system. On the other hand the usual phase space analysis technique has been elegantly used by Yu et al. [18], who analyzed the event of double Hopf bifurcation for such delay dynamical systems. Of course in the literature one can find more interesting and important works done by Elbeyli and Sun [19] and Sinha et al. [20] and their collaborators, which paved different ways for the analysis of such delay differential equations, arising in various problems in engineering.

Our paper is arranged in the following way. The first part involves the formulation of the Galerkin projection technique for the derivation of a finite set of ordinary differential equations whose order depends on the number of shape functions chosen. After having derived these finite set of equations in two different ways (by keeping five and eight shape functions), we explore the dynamics of the system and compare with the results obtained from direct numerical simulation of the delay equations. It is to be mentioned here that the Runge–Kutta integration scheme [21] along with interpolation is not used for the purpose but equations with piecewise constant argument is considered.

In the second part of the paper we have considered the problem of synchronization [22–27] between such a delayed scalar system and a simple one dimensional forced equation. It is now well known that the later class of equations can never show chaos but it has a simple limit cycle due to the forcing. On the other hand the former one though one dimensional exhibits the full potential properties of chaotic dynamics. we show that we can have two kinds of synchronization between two such systems. The first one is when the delayed system drives the ODE and also when the ODE drives the delayed one. In both the cases the onset of synchronization is characterized by the conditional Lyapunov exponent [28,29].

2. Formulation

Delay differential equations or retarded functional differential equations can be written as follows:

$$\dot{x}(t) = f(t, x(t), x(t-\tau_1), x(t-\tau_2), \ldots, x(t-\tau_n)) \tag{1}$$

for $t > 0$ and $x_s(0) = x(-s) = X(s)$, $s \in (0, \tau_n]$. where $X(s)$ is a given initial function and $x_s(t) = x(t-s)$ and n is a finite number with $\tau_n = 1$. The immediate preceding unit interval of time should be kept track during the evolution of $x(t)$. To this end we define a local variable s. In this local interval, the function $x(t)$ coincides with a function $F(t, s)$ given by

$$F(t, s) = x_s(t) = x(t-s) \tag{2}$$

where the delay differential equation governs the evolution of $F(t, s)$ with t. To start with the finite dimensional Galerkin approximation, we approximate the function $F(t, s)$ by