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Nonlinear static analysis of arbitrary quadrilateral plates in very large deflections

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Abstract

Through a linear mapping, an arbitrary quadrilateral plate is transformed into a standard square computational domain in which the deformation and director fields are developed together with the general forms of the uncoupled nonlinear equations. By proper interpolation of displacement and rotation fields on the whole domain, such that the boundary conditions are satisfied, a mathematical model based on the elastic Cosserat theory, is developed to analyze very large deformations of thin plates in nonlinear static loading. The principle of virtual work is exploited to present the weak form of the governing differential equations. The geometric and material tangential stiffness matrices are formed through linearization, and a step by step procedure is presented to complete the method. The validity and the accuracy of the method are illustrated through certain numerical examples and comparison of the results with other researches.

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1. Introduction

Plate structures abound in nature, thin steel plates are widely used as the main structural components of box girders in bridges, plate girders, platforms of offshore structures, shipbuilding and aircraft industries consequently, it is not surprising that these efficient structural forms have been used in many engineering works. Quadrilateral plates are used for a variety of functions in aeronautical and aerospace constructions. In the literature, most nonlinear studies of plates are based on the von-Karman theory [30]. In this study we will use Cosserat theory to analyze very large deformations of thin plates in nonlinear behaviour under static loading. It is well known that a plate is a three-dimensional structure with one dimension, the thickness, small as compared to the remaining two dimensions. Theoretically speaking, two different methods are followed to derive the differential equations governing the nonlinear behavior of plates. In the first method the differential equations and constitutive equations of a three-dimensional continuum are used to expand the position vector of a material point of the deformed body in terms of directors in the direction of the plate thickness [1]. The second method, usually called direct method or Cosserat theory, entails differential geometry and continuum mechanics of a non-Euclidean surface and uses only one director in its formulation [1–5]. This method, in which no a priori approximation is involved, can be argued to be equivalent to a literally infinite series of directors from the stand point of accuracy.

Similar to the analytical approach, two different numerical schemes can also be employed to analyze very large deformations of plates and shells. In the first method a three-dimensional modified element is developed using the three-dimensional differential equations. This approach was initially used by Ahmad et al. [6] in linear analysis of shells. Other researchers extended the use of this element to nonlinear analysis of shells and plates. In particular the works of Hughes and Liu [7,8], and Hughes and Carnoy [9] can be mentioned. Presently this method is recorded as a standard method in finite element textbooks like Bathe [10], Hughes [11] and Belytschko et al. [12]. The second method uses the two-dimensional theory of plates and shells. An approximate formulation of this method was originally developed by Wempner [13] whose article can be regarded as an origin of co-rotational coordinates or co-rotational finite elements in the nonlinear analysis of shells. Along these researches the work of Argyris [14] can also be mentioned. Finite element formulation of Cosserat theory goes back to the first part of a series of papers by Simo and Fox [15]. The latter parts involve linear analysis in small deformations [16], nonlinear analysis in large deformations [17], variation of thickness in nonlinear analysis [18], elastoplastic analysis [19], nonlinear dynamics [20] and finally shell intersections [21]. The work of Wangner [22] on nonlinear shells of revolution can also be mentioned along the above researches.

Co-rotational coordinates or co-rotational finite elements have also found extensive applications in problems in which large displacements are involved along with small strains. Both two- and three-dimensional formulations can be adapted to co-rotational approach. Among researches in which co-rotational formulations are employed, the works of Parisch [23], Buechter and Ramm [24], Sansour and Bufler [25], Peng and Crisfield [26], Jiang and Chernuk [27], Moita and Crisfield [28], and Liu and Hong [29] can be mentioned.

Due to its simpler design from the stand point of constitutive equations, the two-dimensional theory can enhance accuracy in numerical analysis of engineering problems in comparison to three-dimensional methods like three-dimensional modified elements. However, nonlinear