Available online at www.sciencedirect.com

ScienceDirect

Communications in Nonlinear Science and Numerical Simulation 14 (2009) 1008–1012

Communications in Nonlinear Science and Numerical Simulation

www.elsevier.com/locate/cnsns

Short communication

Solution of prey–predator problem by numeric–analytic technique

M.S.H. Chowdhury [a], I. Hashim [a,b,*], S. Mawa [c]

[a] *School of Mathematical Sciences, National University of Malaysia, 43600 Bangi Selangor, Malaysia*
[b] *Institute of Systems Biology, National University of Malaysia, 43600 Bangi Selangor, Malaysia*
[c] *School of Chemical Sciences and Food Technology, National University of Malaysia, 43600 Bangi Selangor, Malaysia*

Received 22 May 2007; accepted 3 November 2007
Available online 22 November 2007

Abstract

In this paper, an analytical expression for the solution of the prey–predator problem by an adaptation of the classical Adomian decomposition method (ADM). The ADM is treated as an algorithm for approximating the solution of the problem in a sequence of time intervals, i.e. the classical ADM is converted into a hybrid numeric–analytic method called the multistage ADM (MADM). Numerical comparisons with the classical ADM, and the classical fourth-order Rungge–Kutta (RK4) methods are presented.

PACS: 02.30.Hq; 02.30.Mv; 02.60.Cb; 02.60.Lj

Keywords: Adomian decomposition method; Non-linear ODEs; Prey–predator problem

1. Introduction

All models of biological systems are essentially based on systems of non-linear ordinary differential equations (ODEs). Both mathematical modelling and simulation are very important in recent studies of biological mathematics. In this analysis, we study the mathematical model of the prey–predator problem in which some rabbits and foxes are considered living together. Foxes eat the rabbits and rabbits eat clover, and when the number of foxes increases, the number of rabbits decreases and the number of foxes decreases, the rabbits will be safe. The relationship of increasing and decreasing in the population of these two kind of animals can be described so-called mathematical model of the problem of prey–predator to the following system of non-linear equations:

* Corresponding author. Address: School of Mathematical Sciences, National University of Malaysia, 43600 Bangi Selangor, Malaysia.
E-mail address: ishak_h@ukm.my (I. Hashim).

1007-5704/$ - see front matter
doi:10.1016/j.cnsns.2007.11.006

$$x' = x(a - by), \tag{1}$$
$$y' = -y(c - dx), \tag{2}$$

where $x(t)$ and $y(t)$ are, respectively, represent the populations of rabbits and the foxes at the time t and a, b, c, d are known coefficients. For more details on the mathematical modelling to the above systems of non-linear equations, we refer the readers to [1]. The problem was solved Biazar and Montazeri [2] using classical Adomian decomposition method and the power series method (PSM) [3].

In this paper, we are interested in the analytic Adomian decomposition method (ADM) [4,5] to solve the prey–predator problems.

Unfortunately, the ADM and PSM are not guaranteed to give analytic solutions valid globally in time as proved by Répaci [6]. This lack of global convergence can, however, be overcome by recursively applying ADM over successive time intervals, as first hinted in [4]. This hybrid numeric–analytic procedure of ADM, we call the multistage ADM (MADM), has been applied to many important equations, such as the multispecies Lotka–Volterra equations [7,8], the extended Lorenz system [9], the chaotic Lorenz system [10–13], systems of ODEs [14], the classical Chen system [15,10] and the Haldane equation for substrate inhibition enzyme kinetics [16]. Similar approach was adopted by Barton et al. [17,18] in solving, respectively, non-stiff and stiff systems of ODEs using Taylor series expansions. We remark that this emerging trend in employing hybrid numeric–analytic methods has also been adopted by Luz Neto et al. [19] in their study of natural convection within porous media via an integral transform method.

In this paper, the total time evolution in the prey–predator problem is simulated by the numeric–analytic MADM. Numerical comparisons with the classical ADM [2] and the classical fourth-order Rungge–Kutta (RK4) methods are presented. In doing so, we correct the results of previous authors [2,3].

2. Solution method

Following [13], we consider the general system

$$X_i' = \sum_{j=1}^{n} a_{ij} X_j + \sum_{p=1}^{n} \sum_{q=1}^{n} a_{ipq} X_p X_q, \quad i = 1, 2, \ldots, n, \tag{3}$$

where the prime denotes differentiation with respect to time. If we denote the linear term (the first term on the r.h.s.) as R_{i1} and the non-linear term (the second term) as R_{i2}, then we can write the above system of equation in the operator form

$$LX_i = R_{i1} + R_{i2}, \quad i = 1, 2, \ldots, n, \tag{4}$$

where L is the differential operator $\mathrm{d}(\cdot)/\mathrm{d}t$. Applying the inverse (integral) operator L^{-1} to (4) we obtain

$$X_i(t) = X_i(t^*) + L^{-1}R_{i1} + L^{-1}R_{i2}, \quad i = 1, 2, \ldots, n. \tag{5}$$

By assuming the general system (3) (or equivalently (4)) is an initial-value problem, it's solution is uniquely determined via the information $X_i(t^*)$ $(i = 1, 2, \ldots, n)$. According to the ADM [4,5], the solution $X_i(t)$ is given by the series

$$X_i(t) = \sum_{m=0}^{\infty} X_{im}(t), \quad i = 1, 2, \ldots, n. \tag{6}$$

Bearing this in mind, the linear term R_{i1} then becomes

$$R_{i1} = \sum_{j=1}^{n} \sum_{m=0}^{\infty} a_{ij} X_{im}, \tag{7}$$

so that $L^{-1}R_{i1}$ is given by

$$L^{-1}R_{i1} = \sum_{j=1}^{n} \sum_{m=0}^{\infty} a_{ij} \int_{t^*}^{t} X_{im}\, \mathrm{d}t, \quad i = 1, 2, \ldots, n. \tag{8}$$