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Short communication

Bit propagation in (2+1)-dimensional systems of coupled sine-Gordon equations

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Abstract

The problem of controlling the propagation of moving breathers in semi-infinite discrete mechanical lattices in two dimensions is tackled here. Nonlinear supratransmission is employed to show that an adequate modulation of the driving amplitude on two adjacent boundaries of the lattice yields an efficient transmission of binary information.

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1. Introduction

As it is known nowadays, the semi-infinite (1+1)-dimensional sine-Gordon equation is able to propagate energy in the form of localized excitations when it is driven harmonically by a frequency in the forbidden band gap and the driving amplitude exceeds a lower threshold [1,2]. This phenomenon is called nonlinear supratransmission, and it has been proved that several nonlinear media present it [3–6]. Many applications based on this nonlinear phenomenon have been proposed [7–10], amongst which the problem of propagating coherent structures is a topic of interest [11,12].

In this work, we study the propagation of localized excitations in discrete semi-infinite systems in two-space dimensions, consisting of coupled sine-Gordon equations that are submitted to harmonic driving in the boundary. We observe that energy propagates into the medium in the form of intrinsic nonlinear modes (also called moving breathers) generated at the origin of our system, and that the nonlinear modes propagate essentially at the same phase velocity, with similar development of amplitude, and carrying identical energetic characteristics. Aided by a standard numerical technique, we will show that a reliable transmission of binary information is achieved by means of an adequate modulation of the driving amplitude in the boundary when the driving frequency takes on values in the forbidden band gap of the medium.

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2. Preliminaries

Let β, γ and J be nonnegative real numbers, and let u be a real function that depends on (x, y, t). Our point of departure in this investigation is the perturbed sine-Gordon equation in two-space dimensions

$$\frac{\partial^2 u}{\partial t^2} - \nabla^2 u - \beta\nabla^2 \frac{\partial u}{\partial t} + \gamma\frac{\partial u}{\partial t} + \sin u = J, \tag{1}$$

defined on the region $(x, y) \in [0, \infty) \times [0, \infty)$ and $t > 0$, and driven harmonically at the boundary by a harmonic perturbation. The model is physically derived from the dynamic description of a Josephson junction of semi-unbounded area with overlap current feed and external magnetic field (see [13, and references therein]); it also appears in the study of spatially localized ultra-short optical pulses in two dimensions, in which the sine-Gordon model is an asymptotic reduction of the two-level Maxwell–Bloch medium [14].

It is important to mention that, due to analytical reasons, the problems studied in Refs. [13,14] do not consider the inclusion of the effects due to the inclusion of nonzero values of β, γ and J. In most realistic studies, however, the inclusion of damping in the model is a task that must be done necessarily. Our model, for instance, includes the presence of external damping through the inclusion of the parameter γ, while the internal damping coefficient β and the parameter J have been included in (1) motivated by the one-dimensional version of this problem, which appears (as slightly modified equations) in the study of long Josephson junctions between superconductors when dissipative effects are taken into account [15], in the study of fluxons in Josephson transmission lines [16], and in the statistical mechanics of nonlinear coherent structures such as solitary waves (see [17, pp. 298–309]).

From a computational point of view, a consistent discretization of (1) is a highly desirable accomplishment. With this goal in mind, we propose now a spatial discretization of the sine-Gordon model which, for the sake of simplicity, we explain in detail for the case when $\beta = \gamma = J = 0$.

Consider the collection of points (m, n), for m and n to nonnegative integers. Let $u_{m,n}$ be a function of the time t, and consider the spatially discrete scheme

$$\frac{d^2 u_{m,n}}{dt^2} - c^2(u_{m+1,n} - 2u_{m,n} + u_{m-1,n}) - c^2(u_{m,n+1} - 2u_{m,n} + u_{m,n-1}) + \sin u_{m,n} = 0. \tag{2}$$

It is important to notice that for c^2 relatively large, the value of $\frac{1}{c^2}$ is relatively small, in which case,

$$\begin{aligned} \frac{\partial^2 u}{\partial x^2}\left(\frac{m}{c^2}, \frac{n}{c^2}, t\right) &\approx \frac{u_{m+1,n}(t) - 2u_{m,n}(t) + u_{m-1,n}(t)}{\frac{1}{c^2}}, \\ \frac{\partial^2 u}{\partial y^2}\left(\frac{m}{c^2}, \frac{n}{c^2}, t\right) &\approx \frac{u_{m,n+1}(t) - 2u_{m,n}(t) + u_{m,n-1}(t)}{\frac{1}{c^2}}. \end{aligned} \tag{3}$$

Indeed, in our study we will consider a discretization of the Laplacian of (1) derived by the discretization proposed in (3), having in mind that such a discretization is a good approximation to the continuous expression of the Laplacian in the limiting-case scenario $c^2 \to \infty$.

2.1. Mathematical model

Let $u_{m,n}$ be a real function of time for every pair of nonnegative integers m and n, and define the operator $\Delta u_{m,n} = u_{m+1,n} + u_{m-1,n} + u_{m,n+1} + u_{m,n-1} - 4u_{m,n}$, for every positive numbers m and n. For such values of m and n, this work studies the problem

$$\begin{aligned} &\frac{d^2 u_{m,n}}{dt^2} - \left(c^2 + \beta\frac{d}{dt}\right)\Delta u_{m,n} + \gamma\frac{du_{m,n}}{dt} + G'(u_{m,n}) = J, \\ &\text{s.t.} \begin{cases} u_{m,n}(0) = \frac{du_{m,n}}{dt}(0) = 0, \\ u_{m,0}(t) = u_{0,n}(t) = \phi(t), \quad \text{with } t > 0. \end{cases} \end{aligned} \tag{4}$$