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Short communication

Electromagnetic force on the wall of cylindrical waveguide

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A B S T R A C T

When transverse electric (TE) wave or transverse magnetic (TM) wave propagates inside a cylindrical waveguide, the electromagnetic force on the wall is investigated. The characteristics of surface charge, current, electric force, magnetic force and electromagnetic force are studied. The results show that the electric force is tension and magnetic force is press. The surface density of electromagnetic force on the wall can be calculated by the difference between magnetic and electric energy density there. For TE wave, the electromagnetic force distribution on the walls may be either tension or pressure in general. However, the electromagnetic force is always pressure for TM wave.

1. Introduction

Electromagnetic force is exerted on the wall of cylindrical waveguide as transverse electric (TE) wave or transverse magnetic (TM) wave travels inside the waveguide. That force may deform the waveguide wall, which will spoil the regular boundary of the electromagnetic field confined in the waveguide. The key parameters, including characteristic function of the mode and cutoff wavelength [1–4], will be distorted.

To avoid that case, it is necessary analyzing the distribution of the electromagnetic force on the wall of the cylindrical waveguide as transverse electric or transverse magnetic wave propagates.

Due to the reason that transverse electromagnetic (TEM) wave never propagate inside any waveguide, the way in Refs. [5,6] cannot be adapted to settle the electromagnetic force distribution on the wall of the cylindrical waveguide. Solving that problem as transverse electric or transverse magnetic wave travels inside is the motivation of this article. On the base of the guided field components, the electric charge and current on the wall are investigated. Then the distribution of the electric force, magnetic force and electromagnetic force on the wall is achieved. The characteristics of these forces are analyzed. The distinction between the force distributions on the wall for propagation of TE and TM wave is presented.

2. TE_{mn} wave

A cylindrical waveguide is straight along Z-direction. Its cross section, a circle of radius R, is sketched in Fig. 1. Hemholts equations hold for electromagnetic waves traveling inside the cylindrical waveguide [7]

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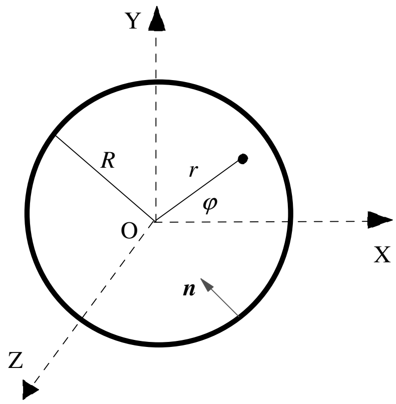

Fig. 1. The cross section of a cylindrical waveguide.

$$\nabla^2 \boldsymbol{E} + k^2 \boldsymbol{E} = 0 \tag{1a}$$

$$\nabla^2 \boldsymbol{H} + k^2 \boldsymbol{H} = 0 \tag{1b}$$

where

$$k^2 = \omega^2 \mu_0 \varepsilon_0 \tag{2}$$

Denote the nth root for the derivation of the Bessel function of the first kind $J_m(z)$ as p'_{mn} [8]. Namely

$$J'_m(p'_{mn}) = 0 \tag{3}$$

With circular boundary, the components of guided TE_{mn} wave in cylindrical coordinates are [9]:

$$H_z = H_0 J_m(k_{cmn} r) \begin{Bmatrix} \cos m\varphi \\ \sin m\varphi \end{Bmatrix} \cdot \cos(\omega t - k_z z) \tag{4a}$$

$$H_r = -\frac{k_z}{k_{cmn}} H_0 J'_m(k_{cmn} r) \begin{Bmatrix} \cos m\varphi \\ \sin m\varphi \end{Bmatrix} \cdot \sin(\omega t - k_z z) \tag{4b}$$

$$H_\varphi = \frac{m k_z}{k_{cmn}^2 r} H_0 J_m(k_{cmn} r) \begin{Bmatrix} \sin m\varphi \\ -\cos m\varphi \end{Bmatrix} \cdot \sin(\omega t - k_z z) \tag{4c}$$

$$E_r = \frac{m k \eta}{k_{cmn}^2 r} H_0 J_m(k_{cmn} r) \begin{Bmatrix} \sin m\varphi \\ -\cos m\varphi \end{Bmatrix} \cdot \sin(\omega t - k_z z) \tag{4d}$$

$$E_\varphi = -\frac{k \eta}{k_{cmn}} H_0 J'_m(k_{cmn} r) \begin{Bmatrix} \cos m\varphi \\ \sin m\varphi \end{Bmatrix} \cdot \sin(\omega t - k_z z) \tag{4e}$$

where

$$k_{cmn} = \frac{p'_{mn}}{R}; \quad k_z = \sqrt{k^2 - k_{cmn}^2}; \quad \eta = \sqrt{\frac{\mu_0}{\varepsilon_0}} \tag{5}$$

The condition for guiding TE_{mn} wave in the waveguide is

$$k > k_{cmn} \tag{6}$$

3. Electromagnetic quantities on the wall

The wall is the boundary of the field propagating inside the waveguide. Substituting $r = R$ as well as Eqs. (5) and (3) into Eq. (4), we obtained the field components on the wall

$$H_z = H_0 J_m(p'_{mn}) \begin{Bmatrix} \cos m\varphi \\ \sin m\varphi \end{Bmatrix} \cdot \cos(\omega t - k_z z) \tag{7a}$$

$$H_r = -\frac{k_z}{k_{cmn}} H_0 J'_m(p'_{mn}) \begin{Bmatrix} \cos m\varphi \\ \sin m\varphi \end{Bmatrix} \cdot \sin(\omega t - k_z z) = 0 \tag{7b}$$

$$H_\varphi = \frac{m k_z R}{p'^2_{mn}} H_0 J_m(p'_{mn}) \begin{Bmatrix} \sin m\varphi \\ -\cos m\varphi \end{Bmatrix} \cdot \sin(\omega t - k_z z) \tag{7c}$$