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Quintic nonpolynomial spline solutions for fourth order two-point boundary value problem

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Abstract

In this paper, we develop quintic nonpolynomial spline methods for the numerical solution of fourth order two-point boundary value problems. Using this spline function a few consistency relations are derived for computing approximations to the solution of the problem. The present approach gives better approximations and generalizes all the existing polynomial spline methods up to order four. This approach has less computational cost. Convergence analysis of these methods is discussed. Two numerical examples are included to illustrate the practical usefulness of our methods.

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1. Introduction

We consider the problem of bending a rectangular simply supported beam of length l resting on an elastic foundation. The vertical deflection w of the beam satisfies the system:

$$w^{(4)} + (\kappa/D)w = D^{-1}r(x), \tag{1.1}$$
$$w(0) = w(l) = w^{(2)}(0) = w^{(2)}(l) = 0, \tag{1.2}$$

where D is the flexural rigidity of the beam, κ the spring constant of the elastic foundation, and the load $r(x)$ acts vertically downwards per unit length of the beam. Mathematically, the system (1.1) and (1.2) belongs to a general class of boundary value problems of the form (see [8,10]):

$$y^{(4)} + f(x)y = g(x), \quad x \in [a,b], \tag{1.3}$$

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subject to the boundary conditions:

$$y(a) - A_1 = y(b) - B_1 = y^{(2)}(a) - A_2 = y^{(2)}(b) - B_2 = 0, \tag{1.4}$$

where A_i, B_i, $i = 1, 2$ are finite real constants. The functions $f(x)$ and $g(x)$ are continuous on the interval $[a, b]$.

The analytical solution of (1.3) subject to (1.4) cannot be obtained for arbitrary choices of $f(x)$ and $g(x)$.

The numerical analysis literature contains other methods developed to find an approximate solution of this problem using polynomial spline functions. Usmani [10] and Usmani and Warsi [11] developed and analyzed second order and fourth order convergent methods for the solution of linear fourth order two-point boundary problem (1.3) subject to (1.4) using quartic, quintic and sextic polynomial spline functions, respectively. Al-Said and Noor [1] and Al-Said et al. [2] demonstrated second order convergent method based on cubic and quartic polynomial spline functions for the solution of fourth order obstacle problems. Usmani [12] established and discussed convergent second order and fourth order methods for this problem with the change in the boundary conditions for first order instead of second order derivatives $y(a) - A_1 = y(b) - B_1 = y^{(1)}(a) - A_2 = y^{(1)}(b) - B_2 = 0$ using quintic and sextic polynomial spline functions, respectively. Also, Rashidinia and Golbabaee [6] and Siddiqi and Akram [7] generated a difference scheme via quintic spline functions for this problem. Zhu [13] introduced optimal quartic spline collocation methods for the numerical solution of this problem based on perturbation technique which gives rise to two optimal quartic spline one step and three step collocation methods. Loghmani and Alavizadeh [3] converted this problems into an optimal control problem and then constructed the approximate solution as a combination of quartic B-splines. Van Daele et al. [5] introduced a new second order method for solving the boundary value problems (1.3) with the boundary conditions involving first derivatives based on nonpolynomial spline function. Siraj et al. [9] solved a system of third order boundary vale problems using nonpolynomial spline functions. Ramadan et al. [4] proposed a second order convergent method for the numerical solution of second order two-point boundary value problems using nonpolynomial spline functions.

The aim of this paper is to construct a new spline method based on a nonpolynomial spline function that has a polynomial part and a trigonometric part to develop numerical methods for obtaining smooth approximations for the solution of the problem (1.3) subject to the boundary conditions (1.4).

The paper is organized as follows: In Section 2, we derive our method. The method is formulated in a matrix form in Section 3. Convergence analysis for second order and fourth order methods is established in Section 4. Numerical results are presented to illustrate the applicability and accuracy in Section 5. Finally, in Section 6, the results of the proposed methods are concluded to illustrate their practical usefulness and accuracy.

2. Derivation of the method

We introduce a finite set of grid points x_i by dividing the interval $[a, b]$ into $(n + 1)$ equal parts where

$$\begin{aligned} &x_i = a + ih, \quad i = 0, 1, \ldots, n, \\ &x_0 = a, \quad x_n = b \quad \text{and} \quad h = \frac{b-a}{n+1}. \end{aligned} \tag{2.1}$$

Let $y(x)$ be the exact solution of the system (1.3) and (1.4) and S_i be an approximation to $y_i = y(x_i)$ obtained by the segment $Q_i(x)$ passing through the points (x_i, S_i) and (x_{i+1}, S_{i+1}).

Each nonpolynomial spline segment $Q_i(x)$ has the form

$$Q_i(x) = a_i \cos k(x - x_i) + b_i \sin k(x - x_i) + c_i(x - x_i)^3 + d_i(x - x_i)^2 + e_i(x - x_i) + f_i, \quad i = 0, 1, \ldots, n. \tag{2.2}$$

where a_i, b_i, c_i, d_i, e_i and f_i are constants and k is the frequency of the trigonometric functions which will be used to raise the accuracy of the method and Eq. (2.2) reduces to quintic polynomial spline function in $[a, b]$ when $k \to 0$. Choosing the spline function in this form will enable us to generalize other existing methods by arbitrary choices of the parameters α, β and γ which will be defined later in the end of this section. Thus, our quintic nonpolynomial spline is now defined by the relations: