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Study of the cloning fidelities for the entanglement of a pair of quantum bits

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Abstract

The cloning fidelities is broadly investigated for the entanglement of a pair of quantum bits. Some new analytical descriptions of the cloning fidelities have been obtained for the entanglement of a pair of quantum bits, generalizing asymmetric cloning transformations and considering a special circumstance. We derive some intact mathematical expressions and simulation results, having the extensive physics meaning, for the investigation of the separability-preserving cloning machine that duplicates all entangled states of two qubits.

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Keywords: The cloning fidelities; Entanglement; Quantum bits; Simulation results

Quantum entanglement is known to be a resource and actualizing much investigation that is central to many quantum information processes such as quantum teleportation, quantum cryptography, or quantum computing [1,2]. Out of these many studies of entanglement, none has so far addressed the issue of whether entanglement can be cloned. In the literature [3], the no-cloning theorem has been stated for the question of cloning quantum states, which precludes the perfect copying of an arbitrary quantum state. The imperfect quantum cloning machines (QCM), involving duplicate with the highest possible fidelity, have been investigated [4]. A large variety of QCMs have been devised, with the purpose of cloning equally well a given set of states in a space of arbitrary dimension [5,6]. Lamoureux and Navez [7] investigating the cloning entanglement of a pair of quantum bits, shown that any quantum operation that perfectly clones the entanglement of all entangled qubit pairs cannot preserve separability, and the entanglement no-cloning principle naturally suggests that some approximate cloning of entanglement is nevertheless allowed by quantum mechanics. Bae and Acn [8] have proved the long-standing conjecture on the equivalence between asymptotic cloning and state estimation. It represents the strongest link between two fundamental no-go theorems of quantum

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mechanics, namely, the impossibilities of perfect cloning and state estimation. Their theoretical results have established scientific values and application prospects for the fundamental research of the separability-preserving optimal cloning machine, duplicating all entangled states of two qubits.

In this latter, we reconsider the cloning entanglement of a pair of quantum bits. We have broadly investigated the cloning fidelities for the entanglement of a pair of quantum bits at the special case. Some new analytical descriptions for the cloning fidelities have been obtained for the entanglement of a pair of quantum bits, generalizing asymmetric cloning transformations and considering a special circumstance. We discover some new mathematical expressions and simulation results, having the extensive physics meaning, for the investigation of the separability-preserving cloning machine that duplicates all entangled states of two qubits. The results present a theoretical basis for the designing efficient quantum information processes by controlling the best information-theoretical use of entanglement.

In their discussions [7], the mathematical description of the cloning transformation is based on the isomorphism between quantum operations and states by using the sufficient condition for covariance [9] and the particular transformation [10]. Imagine that the qubit pair to be cloned is itself entangled with a reference qubit pair. The most general cloning transformation is defined as

$$|\Psi\rangle \rightarrow \sum_{i,j,k,l=0}^{3} s_{ijkl} n_l |i\rangle_a |j\rangle_b |k\rangle_A. \tag{1}$$

The cloning fidelities for the generalized asymmetric cloning transformations ($A \neq B$) and normalization condition have been described by the following mathematical forms:

$$F_a = 4|A|^2 + |B|^2 + |C|^2 + 2Re(AB^* + AC^* + BC^*), \tag{2}$$
$$F_b = |A|^2 + 4|B|^2 + |C|^2 + 2Re(AB^* + AC^* + BC^*), \tag{3}$$
$$4(|A|^2 + |B|^2 + |C|^2) + 2Re(AB^* + AC^* + BC^*) = 1, \tag{4}$$

where the cloning transformation, the sufficient condition for covariance and the generic form of tensor have been expressed by the following forms, respectively:

$$|S\rangle_{R,a,b,A} = \sum_{i,j,k,l=0}^{3} s_{ijkl} |l\rangle_R |i\rangle_a |j\rangle_b |k\rangle_A, \quad |S\rangle_{R,a,b,A} = R^{\otimes 4} |S\rangle_{R,a,b,A}, \quad s_{ijkl} = A\delta_{il}\delta_{jk} + B\delta_{jl}\delta_{ik} + C\delta_{kl}\delta_{ij}.$$

We assume the parameters A, B, and C to be real, and consider the cloning fidelity F_a as an analytical function, contacting with the cloning fidelity F_b and the parameter B. The cloning fidelities for the generalized asymmetric cloning transformations and normalization condition can be transformed as

$$F_a = 4A^2 + B^2 + C^2 + 2(BA + CA + BC), \tag{5}$$
$$F_b = A^2 + 4B^2 + C^2 + 2(BA + CA + BC), \tag{6}$$
$$4(A^2 + B^2 + C^2) + 2(BA + CA + BC) = 1. \tag{7}$$

By using Eqs. (5)–(7), we get new mathematical descriptions for the cloning fidelities of entanglement for the separability-preserving cloning machine that duplicates all entangled states of two qubits. The ulteriorly solutions have been obtained, having following forms as

$$\begin{cases} F_a(B, F_b) = -3B^2 + \frac{F_b + 1}{2} \mp \frac{\sqrt{-3B^2 + F_b} + B}{2} \left(18B^2 - 18B\sqrt{-3B^2 + F_b} - 15F_b + 6\right)^{\frac{1}{2}}, \\ A(B, F_b) = \frac{1}{2}\left(-B - \sqrt{-3B^2 + F_b} \pm \frac{\sqrt{2 + 6B^2 - 5F_b - 6B\sqrt{-3B^2 + F_b}}}{\sqrt{3}}\right), \\ C(B, F_b) = \frac{1}{2}\left(-B - \sqrt{-3B^2 + F_b} \mp \frac{\sqrt{2 + 6B^2 - 5F_b - 6B\sqrt{-3B^2 + F_b}}}{\sqrt{3}}\right), \end{cases} \tag{8}$$