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Nonlinear free vibration analysis of simply supported piezo-laminated plates with random actuation electric potential difference and material properties

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Abstract

Studies are made on nonlinear free vibrations of simply supported piezo-laminated rectangular plates with immovable edges utilizing Kirchoff's hypothesis and von Kármán strain–displacement relations. The effect of random material properties of the base structure and actuation electric potential difference on the nonlinear free vibration of the plate is examined. The study is confined to linear-induced strain in the piezoelectric layer applicable to low electric fields. The von Kármán's large deflection equations for generally laminated elastic plates are derived in terms of stress function and transverse deflection function. A deflection function satisfying the simply supported boundary conditions is assumed and a stress function is then obtained after solving the compatibility equation. Applying the modified Galerkin's method to the governing nonlinear partial differential equations, a modal equation of Duffing's type is obtained. It is solved by exact integration. Monte Carlo simulation has been carried out to examine the response statistics considering the material properties and actuation electric potential difference of the piezoelectric layer as random variables. The extremal values of response are also evaluated utilizing the Convex model as well as the Multivariate method. Results obtained through the different statistical approaches are found to be in good agreement with each other.

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Keywords: Convex model; Duffing's equation; Free vibration; Induced strain actuation; Monte Carlo simulation; Multivariate concept; Piezo-laminated plate; Random variables

1. Introduction

Composite materials are widely used in various types of engineering structures. These materials are in general subject to certain amount of scatter in their elastic moduli due to many factors. Some of these are: misalignments of fibers, fiber pull out, imperfect bonding between the fibers and the matrix, voids, run out, etc

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Nomenclature

a and b	length and width of the plate
$\mathbf{A}$, $\mathbf{B}$ and $\mathbf{D}$	matrices of membrane, coupling and flexural stiffness of the plate
$f(\zeta)$	restoring force function
h	thickness of plate in z direction
h_i	thickness of ith lamina
h_{p}	thickness of piezoelectric layer
$I(\zeta)$	potential energy function
$\mathbf{M}$	moment resultants
$\mathbf{M}_{\Lambda}$	actuator moments
$\mathbf{N}$	stress resultants
$\mathbf{N}_{\Lambda}$	actuator forces
$\mathbf{Q}$	matrix of reduced stiffness coefficients
$\mathbf{Q}_{\mathrm{T}}$	transformed reduced stiffness matrix
t	time
u, v and w	displacements in x, y and z directions
$W_{mn}(t)$	time dependent amplitude
$\boldsymbol{\varepsilon}$	total strain
$\boldsymbol{\varepsilon}^0$	reference plane strain (i.e., strain at $z = 0$)
ζ	amplitude to thickness ratio
ζ_{S}	specified amplitude to thickness ratio
$\boldsymbol{\kappa}$	curvature changes
$\boldsymbol{\Lambda}$	actuation strains
ρ_i	density of ith lamina
$\boldsymbol{\sigma}$	stress
φ	Airy's stress function
$\delta\phi_{\mathrm{a}}$	applied electrical potential difference
ω	nonlinear frequency
ω_{L}	linear frequency

along with variation in the material properties of the matrix and the fiber. Properties of composite materials manufactured by the same process inherently demonstrate differences in their elastic properties. It is very difficult to completely control all the parameters involved in the manufacturing process. For design purposes, it is essential to know the potential variations in the structural response due to the uncertainties in the elastic moduli [1]. This assumes special importance in designs with low margin for error like aerospace applications.

The design margin is small in sensitive applications. Lack of adequate knowledge of the system behavior may result in failure of the design. This has motivated many researchers to study problems with randomness in structural properties. The vibration and buckling response due to variation in the fiber spacing were studied by Leissa and Martin [2]. Shinozuka and Astill [3] obtained statistical properties of eigenvalues of spring supported columns with the support and axial loading along with material and geometric properties as random. Chen and Soroka [4] studied multi-degrees of freedom system with random system properties subjected to deterministic external excitations. Yadav and Verma [5] obtained the buckling response of thin cylindrical shell with random material properties, using classical laminate theory. Second-order response statistics was evaluated by adopting first-order perturbation technique. All the studies of composite structure in random environment are confined to small displacements in the linear domain. Large amplitude dynamic response of composite plates with random material properties has not been addressed fully.

Application of smart structure technologies to aerospace and other systems are expanding rapidly. One of the basic elements of adaptive structures is a thin composite plate with surface bonded or embedded sheet actuators. With tailored laminated plate, induced strain actuation can control its extension, bending and twisting behavior. Plates with distributed-induced strain actuators can be used: to control pointing of preci-