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Comparison of theories for stability of truss structures. Part 1: Computation of critical load

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ABSTRACT

Two largely different theories, i.e. the geometric nonlinear eigenvalue theory and the geometric nonlinear critical point theory, of the stability analysis for truss structures are reviewed by the authors. In this paper, it is pointed out through numerical examples as well as thoroughly theoretical investigations that the eigenvalue theory leads to mistakenly very large solutions of critical load. Though it is correct in theory, the applicability of the critical point theory was inadequately extended to all shallow trusses. To overcome the shortcomings of the stability theories, the authors present two theories of their own with two new approaches for geometric nonlinear analysis and for finding the critical loads for shallow truss structures. Several conclusions are drawn, including: (1) the geometric nonlinear eigenvalue theory is mistaken and (2) the capabilities of various theories are discussed.

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1. Introduction

So far there are four theories have been presented regarding stability problems of truss structures: (1) the geometric nonlinear eigenvalue theory [1,2]; (2) the geometric nonlinear critical point theory [3,4] named by the authors after the word critical point in Ref. [3]; (3) the linear Eulerian theory [5,6]; and (4) the geometric nonlinear Eulerian theory [7,8].

The history of the geometric nonlinear eigenvalue theory can be dated back to long time ago [2] and has been used in many publications e.g. [9–12]. In 1993, the first author used it to compute the critical loads of some examples of truss structure [5]. The results showed that the critical loads were much larger than those obtained by the linear Eulerian theory presented by the first author. It will be analyzed in detail in Section 2 and 3 of the present paper that this theory is mistaken. Similar comment as the abovementioned can also be seen in Ref. [4].

The critical point theory was presented in 1985 and 2000 in Refs. [3,4], respectively. However, it is found that the solution method is too complicated to use in numerical computation. Also, the results of critical loads of a four-bar truss and of a two-bar truss were large and consequently violated the Eulerian stability condition.

The linear Eulerian theory was firstly presented by the first author in 1993 [5]. In this theory, the Euler stability formula from the mechanics of materials is used as the criterion for stability of all the general truss structures. The structural analysis is carried out in the scope of small deformation. In more details, if all members are locally stable the truss is certainly globally stable; the global buckling of the truss is due to the accumulation of local buckling of various members to turn the truss into

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a mechanism. The correctness of this conclusion was proved by theoretical analysis, calculation of practical examples and engineering practice. This theory possesses two objects for the stability analysis of the general trusses: (1) computing the critical load when the sectional areas are given and (2) computing the critical solution of stability, when the applied load is given. The approach for computing the critical load will be described in Section 3 of this paper.

The geometric nonlinear Eulerian theory, on the other hand, uses the Euler stability formula of compressed member for shallow truss structures for which large deformation must be taken into account. In more details, it is the same as the linear Eulerian theory of stability. The approach for finding the critical load will be presented in Section 4 of this paper.

The latter two theories [5–8] are recently proposed by the authors based on the formula of stability of compressed member in strength of materials and are demonstrated being able to conform the requirements of structures and the practice of stability in engineering.

In this paper, the authors intend to reveal the difference, applicability, existing problems as well as the mistake in the four theories, through two aspects of comparisons for the critical load and the critical solution of stability—optimum design of cross-sectional area with stability constraint. Theoretical analysis and comparison of numerical results of examples for critical load in this paper demonstrate that the conventional geometric nonlinear eigenvalue theory is wrong. The differences for critical load among the four theories in some examples, while the applicability and existing problems for them are also provided.

2. Mistake of geometric nonlinear eigenvalue stability theory

2.1. Brief review of the theory

The geometrical nonlinear eigenvalue stability theory can be deduced from the principle of minimum total potential energy under the following basic assumptions:

(1) The geometric equations are derived with small strains and large rotations.
(2) The relations between stresses and strains are linear.
(3) The internal force is uniquely proportional to the external load till the global buckling happens.
(4) The members remain straight lines before the buckling happens.
(5) The buckling is developed by an ultra-large deformation causing the structure to lose its load carrying capacity.
(6) The critical load is not related to the minimum cross-sectional moment of inertia.

With the foregoing assumptions the equilibrium equations are derived

$$(\mathbf{K}_{\mathrm{E}} + \mathbf{K}_{\mathrm{G}})\mathbf{U} = \mathbf{P}, \tag{1}$$

where $\mathbf{U}$ is the vector of nodal displacement, $\mathbf{P}$ is the nodal load vector. $\mathbf{K}_{\mathrm{E}}$ and $\mathbf{K}_{\mathrm{G}}$ are global elastic matrix and geometric matrix of the truss, respectively, defined as

$$\mathbf{K}_{\mathrm{E}} = \sum \mathbf{K}_{\mathrm{E}k}, \quad \mathbf{K}_{G} = \sum \mathbf{K}_{\mathrm{G}k}. \tag{2}$$

For a planar truss, the elementary stiffness matrices are expressed by

$$\begin{aligned} \mathbf{K}_{\mathrm{E}k} &= \frac{E_k A_k}{l_k}\begin{pmatrix} c_k^2 & s_k c_k & -c_k^2 & -s_k c_k \\ s_k c_k & s_k^2 & -s_k c_k & -s_k^2 \\ -c_k^2 & -s_k c_k & c_k^2 & s_k c_k \\ -s_k c_k & -s_k^2 & s_k c_k & s_k^2 \end{pmatrix}, \\ \mathbf{K}_{\mathrm{G}k} &= \frac{N_k}{l_k}\begin{pmatrix} s_k^2 & -s_k c_k & -s_k^2 & s_k c_k \\ -s_k c_k & c_k^2 & s_k c_k & -c_k^2 \\ -s_k^2 & s_k c_k & s_k^2 & -s_k c_k \\ s_k c_k & -c_k^2 & -s_k c_k & c_k^2 \end{pmatrix}, \end{aligned} \tag{3}$$

where $c_k = \cos\alpha_k$, $s_k = \sin\alpha_k$, α_k is the directional angle of member k with respect to x-axis. The axial force N_k is solved though the elementary equilibrium equation

$$\mathbf{K}_{\mathrm{E}}\mathbf{U} = \mathbf{P}. \tag{4}$$

Applying the third basic assumption, i.e. assumption (3) one obtains

$$(\mathbf{K}_{\mathrm{E}} + \lambda\mathbf{K}_{\mathrm{G}})\mathbf{U} = \lambda\mathbf{P}, \tag{5}$$

where λ is the load factor. Eq. (5) yields the well-known eigenvalue equation

$$|\mathbf{K}_{\mathrm{E}} + \lambda\mathbf{K}_{\mathrm{G}}| = 0. \tag{6}$$

By solving Eq. (6), one obtains M eigenvalues for the stability analysis, where M is the number of degree-of-freedom of the truss. Among them the minimum eigenvalue is called the critical load factor, λ_{cr}. The critical load is

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