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Adaptive synchronization of two chaotic systems with stochastic unknown parameters

Hassan Salarieh, Aria Alasty *

Department of Mechanical Engineering, Sharif University of Technology, P.O. Box 11365-9567, Azadi Avenue, Tehran, Iran

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Abstract

Using the Lyapunov stability theory an adaptive control is proposed for chaos synchronization between two different systems which have stochastically time varying unknown coefficients. The stochastic variations of the coefficients about their unknown mean values are modeled through white Gaussian noise produced by the Weiner process. It is shown that using the proposed adaptive control the mean square of synchronization error converges to an arbitrarily small bound around zero. To demonstrate the effectiveness of the proposed technique, it is applied to the Lorenz–Chen and the Chen–Rossler dynamical systems, as some case studies. Simulation results indicate that the proposed adaptive controller has a high performance in synchronization of chaotic systems in noisy environment.

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Keywords: Synchronization; Stochastic chaos; Adaptive control; Lyapunov stability

1. Introduction

In the last few years, synchronization of chaotic dynamical systems has received considerable interest among scientists in various fields [1,2]. The first idea of synchronizing two identical chaotic systems with different initial conditions was introduced by Pecora and Carrols [3–5], and the method was realized in electronic circuits. Synchronization techniques have been improved in recent years, and many different methods have been applied theoretically and experimentally to synchronize the chaotic systems [6–8]. A basic configuration for chaos synchronization is the drive-response pattern, where the response chaotic system must track the drive chaotic trajectory. A number of methods based on this configuration have been proposed. In [9–14] synchronization in hyper-chaotic systems was investigated and a generalized method for synchronization of chaotic systems was proposed [12,13]. Various active and nonlinear control methods were used for chaos synchronization of two identical systems [15–18]. In [19] an adaptive controller for chaos synchronization

* Corresponding author. Tel.: +98 21 6616 5504; fax: +98 21 6600 0021.
E-mail address: aalasti@sharif.edu (A. Alasty).

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of two identical chaotic systems has been proposed. Also synchronization between two different chaotic systems using different nonlinear control schemes has been studied in [20–23]. In [24] the synchronization problem between a delayed chaotic system and a non-chaotic system is studied by means of Galerkin projection method. In [25] adaptive synchronization between two general chaotic systems with affine form was studied. In this work it is assumed that the system unknown parameters are time invariant. An extension to the work of [25] has been presented in [26] where an adaptive control scheme has been proposed for synchronization of two different chaotic systems with time varying unknown parameters. In [26] it has been assumed that the variation of unknown parameters is bounded with known bounds. Using adaptive observer design and Lyapunov stability theorem a nonlinear adaptive synchronization technique was developed for a class of chaotic systems in [27].

All of the mentioned works have modeled the chaotic systems in deterministic form, but in real world due to random uncertainties such as stochastic forces on the physical systems and noisy measurements caused by environmental uncertainties, a stochastic chaotic behavior is produced instead of a deterministic one. In this case the deterministic differential equation of the system must be substituted by a stochastic differential equation. There are a few works in the field of stochastic chaos and its control or synchronization [28–32]. In [28] the stochastic chaos in a Duffing system has been studied where a bounded random process has been added to one of the system parameters, and using the non-feedback control strategy a controller has been designed to quench the chaos in the system. Using feedback control, the synchronization problem of the stochastic Duffing system described in [28] has been investigated in [29]. Ref. [30] has introduced the stochastic chaotic behavior of the brain. A general way to classify stochastic chaos has been presented and applied to population dynamic models in [31]. Synchronization of stochastic delayed neural networks using the Lyapunov control theory and linear matrix inequalities has been investigated in [32]. In [33] a stochastic model of synchronization for chaotic pendulums has been studied, but the system equations are deterministic and have not any white noise excitation. The synchronization of one dimensional diffusion dynamics as an important class of stochastic differential equations has been discussed in [34]. For modeling a stochastic chaotic system, like other standard stochastic differential equations, a white Gaussian noise generated by the derivative of a Wiener process must be applied to a deterministic system. In the case of stochastic chaos, the deterministic system must be chaotic. This modeling provides an Ito stochastic differential form [35]. If the random process which is added to the deterministic system is bounded and is not produced by a Wiener process derivative, the resulted system has not the properties of a stochastic system [36].

This paper is indeed an extension to the works of [25,26]. The synchronization problem of two general chaotic systems whose coefficients are unknown and stochastically time varying is studied. Stochastic behavior of the parameters is modeled through white Gaussian noise generated by a Wiener process; hence the resulted systems are standard stochastic differential equations. It is shown that there is not any Markov control law to synchronize the chaotic systems completely. Using the Lyapunov stability theory a Markovian adaptive control law is designed for synchronizing the stochastic chaotic behavior of the two systems, in the sense of mean square convergence. It is proved that by applying the proposed control scheme, the variance of the synchronization error can converge to any arbitrarily small bound around zero. As two cases of study the synchronization problems of the Lorenz–Chen and the Chen–Rossler, as the drive-response systems, are studied by using the presented method. Simulation results show that the proposed method can be successfully used in synchronization of chaotic systems which have noisy parameters.

2. Problem statement

It is assumed that the dynamics of the drive chaotic system is given by

$$\dot{x} = f(x) + F(x)\theta, \tag{1}$$

where $x \in \Re^n$ is the state vector of the system, $\theta \in \Re^m$ is the vector of the system parameters, $f \in C^1(\Re^n, \Re^n)$, $F \in C^1(\Re^n, \Re^{n\times m})$ and for all x and t, $\|F(x,t)\|_2 < N_F$ where N_F is a known positive constant, i.e. F is bounded, and $\|\cdot\|_2$ is the Euclidian norm of matrices and vectors [37]. The system parameters deviates randomly around their unknown average values, $\bar{\theta}$: