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Global robust stability criteria of stochastic Cohen–Grossberg neural networks with discrete and distributed time-varying delays

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Abstract

The paper is concerned with the problem of robust asymptotic stability analysis of stochastic Cohen–Grossberg neural networks with discrete and distributed time-varying delays. Based on the Lyapunov stability theory and linear matrix inequality (LMI) technology, some sufficient conditions are derived to ensure the global robust convergence of the equilibrium point. The proposed conditions can be checked easily by LMI Control Toolbox in Matlab. Furthermore, all the results are obtained under mild conditions, assuming neither differentiability nor strict monotonicity for activation function. A numerical example is given to demonstrate the effectiveness of our results.

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Keywords: Stochastic Cohen–Grossberg neural networks; Delay-dependent robust stability; Discrete and distributed time-varying delays; Norm-bounded uncertainties

1. Introduction

In the past few years, neural networks such as Hopfield neural networks [1], cellular neural networks [2], and bi-directional associative memory neural networks [3] have attracted the attention of many scientists. The Cohen–Grossberg neural networks (CGNN) were first proposed by Cohen and Grossberg [4]. As is well known, Cohen–Grossberg neural networks include many models from different research fields, such as neurobiology, population biology and evolutionary theory. The stability of neural networks plays an important role in their potential applications, such as associative content addressable memories, pattern recognition and optimization, so it is of significance and necessary to investigate the stability of CGNN.

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It is well known that time delays may occur in neural processing and signal transmission, which can cause instability and oscillations in system. On the other hand, neural networks usually have a spatial extent due to the presence of a multitude of parallel pathways with a variety of axon sizes and lengths, and hence that there is a distribution of propagation delays over a period of time. So the distributed delays should be incorporated in the model. In other words, it is often the case that the neural networks model possesses both discrete and distributed delays. As for the CGNN, due to its generality and wide prospect of applications, a number of sufficient conditions have been proposed, see for [5–11] the exponential stability, [12–17] for global asymptotic stability and [18–20] for distributed delays and the references therein.

When performing the computation, there are many stochastic perturbations that affect the stability of neural networks. A neural network could be stabilized or destabilized by certain stochastic inputs. It implies that the stability analysis of stochastic neural networks also has primary significance in the research of neural networks. Recently, there are some research issues about stochastic neural networks, see [21–24] and references therein. In [21,22], the stability problem is investigated for Hopfield stochastic neural networks. In [23], the authors obtain some sufficient criteria ensuring the almost sure exponential stability of the stochastic Cohen–Grossberg neural networks by constructing suitable Lyapunov functional and employing the semimartingale convergence theorem. However, the condition does not consider the entries of the connection matrices. Thus the difference between the neuron excitatory and inhibitory effects might be ignored. The global asymptotic stability analysis problem is considered for a class of stochastic Cohen–Grossberg neural networks with constant mixed delays in [24]. The proposed criterion is dependent of the bound parameters of amplification function. And the criterion is delay-independent on discrete delay and it doesn't consider the uncertainties.

In our paper, we develop new delay-dependent robust stability conditions for stochastic Cohen–Grossberg neural networks with discrete and distributed time-varying delays by utilizing Lyapunov functions. The activation function is vary general, assuming neither differentiability nor strict monotonicity. The stability criteria are derived as LMIs which can be efficiently solved by the LMI Control Toolbox in Matlab. The effectiveness of the proposed stability criteria is illustrated in a numerical example.

Notations 1. Let $(\Omega, \mathscr{F}, \{\mathscr{F}_t\}_{t\geqslant 0}, P)$ be a complete probability space with a filtration $\{\mathscr{F}_t\}_{t\geqslant 0}$ satisfying the usual conditions (i.e. it is right continuous and $\mathscr{F}_0$ contains all P-null sets). The mathematical expectation operator with respect to the given probability measure P is denoted by $\mathbf{E}\{\cdot\}$.

2. System description

The delayed stochastic Cohen–Grossberg neural networks can be described by the following delay differential equation:

$$\begin{aligned} \mathrm{d}x(t) = & \left\{-\alpha(x(t))[\beta(x(t)) - Af_1(x(t)) - Bf_2(x(t-h(t))) - C\int_{t-\tau(t)}^{t} f_3(x(s))\mathrm{d}s]\right\}\mathrm{d}t \\ & + \sigma(t, x(t), x(t-h(t)))\mathrm{d}\omega(t), \end{aligned} \tag{1}$$

where $x(t) = [x_1(t), \ldots, x_n(t)]^{\mathrm{T}} \in R^n$ is the state variable, $\alpha(x(t)) = \mathrm{diag}\{\alpha_1(x_1(t)), \ldots, \alpha_n(x_n(t))\}$ represents an amplification function and assumed to be positive, bounded and locally Lipschitz continuous, $\beta(x(t)) = [\beta_1(x_1(t)), \ldots, \beta_n(x_n(t))]^{\mathrm{T}}$ is the behaved function. $f_i(x) = [f_{i1}(x_1(t)), \ldots, f_{in}(x_n(t))]^{\mathrm{T}} (i = 1,2,3)$ are the neuron activation function. A is the feedback matrix, B and C represent the discretely delayed connection weight matrix and the distributive delayed connection weight matrix, respectively. The discrete and distributed time delays are time-varying and satisfy $0 \leqslant h(t) \leqslant h$ and $\dot{h}(t) \leqslant \mu < 1$, $0 \leqslant \tau(t) \leqslant \tau$. $\omega(t) = [\omega_1(t), \ldots, \omega_m(t)]^{\mathrm{T}} \in R^m$ is a m-dimensional Brownian motion defined on a complete probability space $(\Omega, \mathscr{F}, \{\mathscr{F}_t\}_{t\geqslant 0}, P)$. Moreover, σ satisfies

$$\mathrm{trace}[\sigma^{\mathrm{T}}(t, x(t), x(t-h(t)))\sigma(t, x(t), x(t-h(t)))] \leqslant x^{\mathrm{T}}(t)\Sigma_1^{\mathrm{T}}\Sigma_1 x(t) + x^{\mathrm{T}}(t-h(t))\Sigma_2^{\mathrm{T}}\Sigma_2 x(t-h(t)). \tag{2}$$

Assumption 1. $\beta_i(x): R \to R$ is continuous and differentiable, and

$$\beta_i'(x) \geqslant \gamma_i > 0 \ \forall x \in R, i = 1, \ldots, n. \tag{3}$$