Available online at www.sciencedirect.com

ScienceDirect

Communications in Nonlinear Science and Numerical Simulation 12 (2007) 243–253

Communications in Nonlinear Science and Numerical Simulation

www.elsevier.com/locate/cnsns

A class of subgrid-scale models preserving the symmetry group of Navier–Stokes equations

Dina Razafindralandy *, Aziz Hamdouni, Claudine Béghein

L.E.P.T.A.B., Université de La Rochelle, Avenue Michel Crépeau, 17042 La Rochelle Cedex 01, France

Received 7 January 2005; received in revised form 24 February 2005; accepted 25 February 2005
Available online 11 April 2005

Abstract

Navier–Stokes equations (NS) admit transformations which transform a solution to another solution (galilean transformation, scaling transformation, ...). They also admit viscosity dependent transformations which transform a solution to a solution of another NS with different viscosity. These particular transformations are called symmetries of NS. Each of them has a physical role (such as conservation laws, ...). A consistent turbulence model should then remain invariant under these symmetry transformations. Unfortunately, this is not the case of several models.

In this article, a class of subgrid-scale models preserving the symmetries of NS is built. This class is then refined such that the models respect the second law of thermodynamics. One of the simplest models of the class is tested to the flow in a ventilated room. Better results than those provided by Smagorinsky and dynamic models are obtained.

PACS: 47.27.Eq

Keywords: Turbulence modeling; Large-eddy simulation; Symmetry group

* Corresponding author. Tel.: +33 5 4645 8332; fax: +33 5 4645 8241.
E-mail addresses: drazafin@univ-lr.fr (D. Razafindralandy), ahamdoun@univ-lr.fr (A. Hamdouni), cbeghein@univ-lr.fr (C. Béghein).

1007-5704/$ - see front matter
doi:10.1016/j.cnsns.2005.02.006

1. Introduction

Thanks to Lie theory, Pukhnachev [1] could calculate symmetries of Navier–Stokes equations (NS), i.e. transformations in the space of variables

$$t, x, u, p$$

which transform a solution to another solution. Ibragimov and Ünal [2] completed these symmetries by calculating transformations in the space variables

$$t, x, u, p, \nu$$

which transform a solution of NS to a solution of another NS with different viscosity. The set of all these symmetry transformation is called the symmetry group of NS. This group plays an important role in the description of physical phenomena (conservation laws, scaling laws, ...). A consistent model of turbulence must then preserve it. Unfortunately, as shown by Oberlack [3], several models in the literature violate this property. In addition, among the standard models, many do not satisfy the second law of thermodynamics because they induce negative dissipation. When it is possible, an a posteriori forcing becomes then unavoidable.

This paper aims to build and validate models which both preserve the symmetry group of NS and naturally respect the second law of thermodynamics. It is structured as follows. In Section 2, a remind of Lie theory and the symmetries of NS is done. Next, a class of models preserving the symmetry group is built in Section 3. This class is then refined in Section 4 such that the models conform with the second law of thermodynamics. Finally, a simple model of the class is tested in Section 5 and compared with Smagorinsky and dynamic models (see [4,5]).

2. The symmetry group of Navier–Stokes equations

Consider an incompressible newtonian fluid, with density ρ and kinematic viscosity ν. The motion of this fluid is governed by the Navier–Stokes equations

$$\begin{cases} \dfrac{\partial u}{\partial t} + \operatorname{div}(u \otimes u) + \dfrac{1}{\rho} \nabla p = \operatorname{div} \tau \\ \operatorname{div} u = 0 \end{cases} \tag{1}$$

where τ is the viscous constraint tensor,[1] which can be linked to the strain rate tensor $S = (\nabla u + {}^T\nabla u)/2$ according to the relation

$$\tau = \frac{\partial \psi}{\partial S},$$

ψ being a positive and convex "potential" defined by

$$\psi = \nu \operatorname{tr} S^2.$$

Let $y = (t, x, u, p)$. To simplify, we designate (1) by

$$\mathscr{E}(y) = 0. \tag{2}$$

[1] In fact, it is an abusive language because the real constraint is $\rho\tau$.