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Transitions from bursting to spiking due to depolarizing current in the Chay neuronal model

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Abstract

Distinct transitions of firing activities from bursting to spiking induced by the depolarizing current I are explored near the Hopf bifurcations in the Chay neuronal system. The period-1 "circle/homoclinic" bursting at one rest state makes a transition slowly to repetitive spiking with the parameter I increasing. However, the "Hopf/homoclinic" bursting via a "fold/homoclinic" hysteresis loop at another rest state may transit to continuous spiking abruptly by increasing I.

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1. Introduction

Neuronal firing activities are of fundamental importance to reveal the mechanism of neuronal coding. Many different dynamical models have been studied to explore their dynamical behavior, among which bursting is the most important one and has been investigated in many experiments

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[1–4] and theoretical studies [5–12]. Bursting means that the firing activity of neuron alternates between a rest state and repetitive spiking (that is, a burst). Many mathematical models of neuronal bursters can be written in the following singular perturbation form [11]:

$$\dot{x} = f(x, u), \tag{1}$$

$$\dot{u} = \mu g(x, u), \tag{2}$$

where $\mu \ll 1$ represents the ratio between the fast and slow time scales. $x \in \mathbf{R}^m$ is the fast variable responsible for repetitive spiking, and $u \in \mathbf{R}$ is the slow variable to modulate spiking. When considering the fast subsystem $\dot{x} = f(x, u)$ with the one-dimensional slow variable u as a time-dependent bifurcation parameter, there are two important kinds of bifurcations of the fast subsystem associated with bursting: the bifurcation of a rest state leading to repetitive spiking and that of repetitive spiking leading to a rest state. These bifurcations of the fast subsystem can reveal explicitly how the spike-generating mechanism interacts with the slow dynamics to produce bursting behavior and provide a complete classification scheme for bursting, see Table 4 in [11].

In contrast with bursting, periodic spiking only corresponds to the existence of a large stable limit cycle and always lies near this limit cycle.

Many excitable cells at rest can exhibit interesting discharge activities in response to various external stimuli. As a consequence, the firing patterns induced by depolarizing current, as well as their dynamical behavior, are studied through the Chay system by means of numerical simulation and fast/slow dynamic analysis here. This paper is organized as follows. Section 2 presents the Chay model with depolarizing current. Transition phenomena from distinct types of bursting to spiking induced by the depolarizing current are discussed in Section 3. Finally, a conclusion is given in Section 4.

2. Chay model with depolarizing current

The following three equations form the Chay model with depolarizing current [6]:

$$\frac{\mathrm{d}V}{\mathrm{d}t} = g_I^* m_\infty^3 h_\infty (V_I - V) + g_{K,V}^* (V_K - V) n^4 + g_{K,C}^* \frac{C}{1 + C} (V_K - V) + g_L^* (V_L - V) + I, \tag{3}$$

$$\frac{\mathrm{d}n}{\mathrm{d}t} = \frac{n_\infty - n}{\tau_n}, \tag{4}$$

$$\frac{\mathrm{d}C}{\mathrm{d}t} = \rho [m_\infty^3 h_\infty (V_C - V) - k_C C], \tag{5}$$

where three dynamical variables are V (the membrane potential), n (the probability of opening the voltage-sensitive K^+ channel) and C (the intracellular concentration of Ca^{2+} ions). V_K, V_C are the reversal potentials for K^+ and Ca^{2+} channels, respectively, and V_L, V_I are that for other ionic channels. The explicit expressions for m_∞, h_∞ and n_∞ can be written generally as $y_\infty = \alpha_y/(\alpha_y + \beta_y)$, where y represents m, n or h, with