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Symmetry-breaking bifurcation analysis of stochastic van der pol system via Chebyshev polynomial approximation

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Abstract

Chebyshev polynomial approximation is applied to the symmetry-breaking bifurcation problem of a stochastic van der Pol system with bounded random parameter subjected to harmonic excitation. The stochastic system is reduced into an equivalent deterministic system, of which the responses can be obtained by numerical methods. Nonlinear dynamical behaviors related to various forms of stochastic bifurcations in stochastic system are explored and studied numerically.

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Keywords: Chebyshev polynomial; Stochastic van der Pol system; Symmetry-breaking bifurcation; Probability density function

1. Introduction

A stochastic system, we consider here, means a system with random parameters, which are typically bounded in nature. And a type of stochastic bifurcation means bifurcation in a stochastic

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nonlinear system under harmonic excitations, which differs from another type of stochastic bifurcation resulting form a deterministic nonlinear system under a harmonic excitation, together with a random excitation. There are several kinds of mathematical methods available for the stochastic structure analysis: the Monte–Carlo method [1,2], the stochastic perturbation method [3–5], and the orthogonal polynomial approximation method introduced by Spanos and Ghanem [6], Jonsen and Iwan [7], and Li [8,9]. The Monte–Carlo method is simple and universal, but usually involved with a quite amount of computational effort. The stochastic perturbation method is involved with the least computational effort, but usually restricted to system with random variables of small fluctuations only. The orthogonal polynomial approximation method does not require the small random perturbation assumption, thus providing more applicability. Recently, Fang, Leng [10,11] firstly applied Chebyshev polynomial approximation to the evolutionary random response problem of a linear stochastic system, and then further to study bifurcation and chaos of stochastic Duffing system.

The van der Pol equation arises as a typical model of self-excited oscillators. The periodically forced oscillator [12]

$$\ddot{x} + ax^2\dot{x} - \bar{b}\dot{x} + x = F\cos\omega t$$

provides a prototype equation of a periodically disturbed limit cycle that shows a large variety of nonlinear phenomena, among them saddle-node bifurcation, symmetry-breaking bifurcation, period-doubling bifurcation and chaos. Much work has already been done to investigate the properties of this system experimentally [13,14] and theoretically [15]. In particular, the case of large damping has been treated by many authors, and important analytical results and concepts of complex dynamics have been derived [16–18]. Stability and bifurcation studies have been carried out [12,19–21]. Numerical investigations have provided example of devil's staircases, period-doubling cascades, and chaotic attractors [22,23]. The global bifurcation characteristics of this system have already been studied [24,25]. However, these results are restricted to deterministic van der Pol system.

In this paper, the main attention is to analyze symmetry-breaking bifurcation phenomena of van der Pol system under harmonic excitation with random bounded parameter using the Chebyshev polynomial approximation. The equivalent deterministic system for the stochastic van der Pol system is derived, and the responses are obtained by numerical methods. Considering ω, F as the bifurcation parameters respectively, we explore symmetry-breaking bifurcation systematically in the stochastic van der Pol system.

2. Chebyshev polynomial

Random parameters for engineering structures are bounded in nature. One of the reasonable probability density function (PDF) models for the bounded random variables is the arch-like PDF, which is shown in Fig. 1 and can be expressed as follows [10]:

$$p(u) = \begin{cases} (2/\pi)\sqrt{1-u^2} & \text{as } |u| \leqslant 1, \\ 0 & \text{as } |u| > 1. \end{cases} \tag{1}$$