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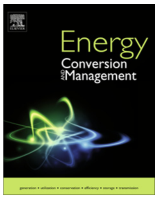

Unity maximum transient energy growth of heat-driven acoustic oscillations

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ABSTRACT

Transient energy growth of acoustic disturbances may trigger thermoacoustic instability in a non-normal thermoacoustic system. In this work, minimizing transient energy growth of heat-driven acoustic oscillations in an open-ended thermoacoustic system is considered. For this, a state-space thermoacoustic model with an acoustically compact heat source and distributed monopole-like actuators is developed. The heat source gives rise to the mean temperature jump, as experimentally measured. It is modeled with a modified King's Law. Coupling the unsteady heat release model with a Galerkin series expansion of the acoustic waves present enables the time evolution of flow disturbances and acoustical energy to be calculated, thus providing a platform on which to gain insight on the system's transient stability behaviors and the non-normal response of the system to the dynamic actuators. It is first shown that implementing a linear-quadratic regulator (LQR) leads to the system being asymptotically stabilized. However, the LQR optimization strategy fails in eliminating the transient growth. This finding is consistent with Pseudospectra analysis of the present system. In order to achieve unity maximum transient growth, a Lyapunov-based optimization strategy is systematically designed. It is found that this optimization strategy achieves both exponential decay of the acoustical energy and unity maximum transient growth. Furthermore, the sound pressure level is reduced by approximately 25 dB. In addition, the number of the actuators K is shown to be related to the mode number N as $K = N$.

1. Introduction

Self-sustained heat-driven acoustic oscillations are wanted in thermoacoustic heating or cooling systems [1–6,47]. However, such oscillations are undesirable in many combustion systems, such as aero-engine afterburners, rocket motors, ramjets, boilers and furnaces [9,10]. The oscillations are also known as thermoacoustic instability. It is generated by a dynamic interaction between unsteady heat release and flow disturbances present [11–15]. When unsteady heat is added in phase with the pressure oscillations [16], the energy of acoustic disturbances increases. Unsteady heat release is a monopole-like sound source to produce acoustic waves [17]. The sound waves [1,2,18] propagate within the system and partially reflect from boundaries to arrive back at the heat source due to impedance change [47]. And more unsteady heat release may be caused under certain conditions. This interaction between unsteady heat release and acoustic disturbances may result in large-amplitude and damaging self-sustained pressure oscillations [19] (also known as thermoacoustic instability). Such oscillations can become so intense that they cause overheating, flame flashback or blow-off, structural vibration and costly mission failure [11,20].

Thermoacoustic instability is currently one of the major challenges for land-based gas turbine and aero-engine manufacturers [21–23]. To mitigate thermoacoustic instability, there are two typical approaches, which break the coupling between the unsteady heat release and acoustic waves. One is passive and the other is active/feedback control [18]. Passive control [24] involves redesigning the ignition system or changing operating conditions or applying Helmholtz resonators [25] and acoustic liners to increase acoustic damping/loss. Applying such passive approach in practical engine systems is well-reviewed [24]. Passive approach is low-cost and simple. However, it cannot respond to changes in operating conditions due to the lack of dynamic actuators. However, feedback control approach can be implemented to various types engine systems and applied to a wide range of operation.

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Nomenclature

$\mathcal{C}_1, \mathcal{C}_2$	damping coefficients in Eq. (16)
c	speed of sound in air, m/s
c_p, c_v	heat capacity ratio at constant pressure and volume, kJ/kg K
d_w	the diameter of the heated wire, m
E	acoustical energy, J
G	transient growth
$G_{\max}$	maximum transient growth
k	kth eigenmode,
K	heat conductivity, W/mK
L_w	total length of the heated wire, m
L	total length of the combustor, m
$\overline{M}$	mean flow Mach number
N	number of eigenmodes,
$\bar{p}$	mean pressure, Pa
p'	pressure fluctuation, Pa
$\dot{Q}_a$	actuation signal in Eq. (3), J/m^3
$\dot{Q}_s$	unsteady heat release rate, KJ/s
$\mathbf{Q}$	matrix defined in Eq. (26)
R_k, S_k	actuation parameters in Eq. (5)
S	cross-sectional area of the combustor, m^2
S_{ak}	cross-sectional area of the kth actuator, m^2
t	time, s
T	Temperature, K
T_w	Heated wire temperature, K
$\mathbf{u}_m$	control input in Eq. (15)
$\bar{u}, u'$	mean flow velocity and fluctuating part, m/s
$\mathcal{V}$	Lyapunov function defined in Eq. (31)
x	axial location along the combustor, m
x_f	axial distance of the heated wires, m

Greek symbols

α_n, β_n	coefficients defined in Eq. (11)
α_{ak}	cross-sectional area ratio
δ	Kronecker delta function
η	time-varying function as defined in Eq. (6)
γ	the ratio of specific heat
κ_n	coefficient defined in Eq. (6)
λ	conduction coefficient, W/mK
μ	viscosity, kg/m s
ω	oscillation frequency, rad/s
ψ	basis function as defined in Eq. (6)
ρ	air density, kg/m^3
τ	time delay, s
υ_{ak}	the kth actuation signal
ζ	damping coefficient as defined in Eq. (16)

Subscript

1,	pre-heating
2,	after-heating
a,	actuator

Superscript

~	instantaneous value
˙	time derivative
¯	mean value

Typically, active control [19,26,27] is implemented in closed-loop configuration. The active controller drives a dynamic actuator in response to a sensor measurement. One of the general actuation actions is to modulate the acoustic field by using a monopole-like sound source such as a loudspeaker [28]. Experimental investigation of using loudspeakers to mitigate thermoacoustic instabilities was conducted by Campos-Delgado et al. [29] on a non-premixed spray combustor via implementing linear quadratic Gaussian (LQG)/loop transfer recovery (LTR) and H_∞ loop-shaping. Comparison was then made by evaluating the performance of these controllers and the conventional phase-shifted controller. It is found that the phase-shifted controller resulted in worse attenuation of the pressure oscillations than LQG/LTR or H_∞.

Another general actuation is to modulate the unsteady heat release rate by using a secondary fuel injector [30,31]. Bernier et al. [31] applied a secondary fuel injector to stabilize a lean premixed pre-vaporized swirl-involved combustor. The combined transfer function of the burner and the actuation system were measured by using two methods. Insightful reviews of thermoacosutic instability and its feedback control were reported by McManus et al. [19]. The objective of traditional linear controllers applied on a thermoacoustic system is to make all the eigenmodes decay exponentially, i.e. to make the system stable under classical linear stability. However, when the thermoacoustic eigenmodes are non-orthogonal as typically found in a practical system, controlling the dominant eigenmode alone may cause other modes being excited due to the coupling effect [32].

Non-orthogonality of thermoacoustic eigenmodes (i.e., non-normality) has received more attentions recently [33–35]. It has been shown that in a linearly stable but non-normal system (characterized with non-orthogonal eigenmodes) [8,34,35] there can be significant transient energy growth of small-amplitude acoustic perturbations. It has also been shown [33] that the non-orthogonality results from the presence of unsteady heat release or complex impedance boundary conditions. When the transient growth of acoustic disturbances is large enough, thermoacoustic instability might be triggered. Experimental measurement of the transient growth was performed on a lean-premixed gas turbine combustor in Cambridge [22].

Transient growth of acoustic disturbances cannot be predicted by such classical linear stability theory [28], since it provides information only about the long-term evolution of the eigenmodes [35]. Similarly, implementing conventional linear controllers might be associated with transient growth. Kulkarni et al. [35] might be the first group of researchers studying the effect of non-normality on the performance of such linear controllers on stabilizing a Rijke-type thermoacoustic system [36–40]. Theoretical analysis was conducted by expanding the acoustic perturbation via Galerkin technique. It was claimed by Kulkarni et al. [35] that the eigenfunction $\psi(x) = \mathcal{F}(j\pi x)$ representing the mode-shape is the same as that when there is no mean temperature. This finding is different from those obtained from the previous works [41,44].

The implementation of conventional linear controller such as pole-placement [35] or LQR [44] were shown to lead to the thermoacoustic system being non-normal. And these controllers failed in preventing the ‘nonlinear driving’ of low amplitude acoustic disturbances by controlling their transient growth. For this, so-call ‘transient growth controllers’ were developed [35,44]. It is based on the critical condition derived by Whidborne and McKernan [42]. Kulkarni et al. [35] showed that the pole-placement controller needs 6 actuators to minimize transient growth in a system with 3 modes. However, neither the ‘transient growth controller’ nor the relationship between the actuators and eigenmodes was systematically designed. Furthermore, the values of the distributed