Available online at www.sciencedirect.com

ScienceDirect

Communications in Nonlinear Science and Numerical Simulation

Communications in Nonlinear Science and Numerical Simulation 12 (2007) 1454–1463

www.elsevier.com/locate/cnsns

Influence of variable heat flux on natural convection along a corrugated wall in porous media

Shalini [a,b], B.V. Rathish Kumar [a,*]

[a] Department of Mathematics, Indian Institute of Technology Kanpur, 208 016 UP, India

[b] INRIA Rocquencourt, BP 105, 78153 Le Chesnay Cedex, France

Received 19 January 2005; received in revised form 12 January 2006; accepted 5 February 2006
Available online 31 March 2006

Abstract

In this work the coupled non-linear partial differential equations, governing the free convection from a wavy vertical wall under a power law heat flux condition, are solved numerically. For both Darcy and Forchheimer extended non-Darcy models, a wavy to flat surface transformation is applied and the governing equations are reduced to boundary layer equations. A finite difference scheme based on the Keller Box approach has been used in conjunction with a block tri-diagonal solver for obtaining the solution. Detailed simulations are carried out to investigate the effect of varying parameters such as power law heat flux exponent m, wavelength–amplitude ratio a and the transformed Grashof number Gr'. Both surface undulations and inertial forces increase the temperature of the vertical surface while increasing m reduces it. The wavy pattern observed in surface temperature plots, become more prominent with increasing m or a but reduces as Gr' increases.

PACS: 47.11.+j

Keywords: Natural convection; Corrugated surface; Variable heat flux; Non-similarity technique; Keller Box method

1. Introduction

The study of heat transfer in a fluid-saturated porous medium has a wide range of applications in the engineering and physical problems, some of which include the spreading of pollutants, water movement in reservoirs, thermal insulation engineering, building science and convection in the earth's crust etc. Motivated by several engineering and real life applications, a large number of problems have been solved with a variety of heating conditions. In a fluid-saturated porous media if a surface, having the temperature different from the ambient porous medium is inserted or when the temperature of any of the bounding surface is changed suddenly, natural convection takes place due to the spatial variation of temperature/density. Several studies [1–4] have been done to understand the convection process under the influence of a heated surface having

* Corresponding author. Tel.: +91 512 2597660; fax: +91 512 2597500.
E-mail address: bvrk@iitk.ac.in (B.V. Rathish Kumar).

1007-5704/$ - see front matter
doi:10.1016/j.cnsns.2006.02.003

uniform temperature. By using either analytical or numerical approaches, Bejan [5], Rees and Pop [6], Kumari et al. [7], Prasad and Kulacki [8], Rathish Kumar [9] etc. solved the natural convection problem in a fluid-saturated porous media with uniform heat flux condition. Moulic and Yao [10] studied the free convection process in continuum fluid subjected to the constant wall heat flux condition. However, in view of its significance, variable wall temperature (VWT) or variable heat flux (VHF) conditions are equally important to analyze. Few authors, such as Hsieh et al. [11,12] used non-similarity transformation to solve the problem of mixed convection along a vertical flat plate under VHF and VWT conditions. Later on, Aldoss et al. [13] examined the mixed convection process by considering a non-Darcy model under both the heating conditions. Hung et al. [14] used VWT condition and analyzed non-Darcian natural convection along a flat vertical surface.

Surface non-uniformities are often encountered in many engineering problems. Sometimes surfaces are intentionally roughened to enhance the heat transfer [15]. Examples of the surface non-uniformities are encountered in cavity wall insulating system, grain-storage containers, in flat plate condensers in refrigerators etc. A review of the natural convection heat transfer in continuum fluid along the non-uniform surfaces is given by Bhavnani and Bergles [16]. In view of the complex geometries of such surfaces it is very difficult to determine the flow field and the associated heat transfer characteristics. In the context of convection process in the continuum fluid along a sinusoidal wavy surface, Yao [17] presented a boundary layer transformation to convert the non-uniform surface to a flat surface. Rees and Pop [6,18] used this approach and solved the non-similar boundary layer equations governing the free convection process in a fluid-saturated porous media along a sinusoidal wavy surface under Darcy flow assumptions.

Darcy's law states that the volume-averaged velocity is proportional to the pressure gradient. It is only suitable for the case of low flow velocity. To model a physical situation with high flow rate near the wall, it is necessary to include non-Darcian effects in the analysis of convection process in porous medium. Plumb and Huenefeld [19], Bejan and Poulikakos [20], Kumari et al. [21], Rees and Pop [22], Aldoss et al. [13] and Hung et al. [14] etc., have analyzed the convection process under different wall heating conditions using non-Darcian model. In the present work we aim to analyze the free convection process in a fluid-saturated porous media along a sinusoidal wavy surface under variable heat flux condition. We assume that the heat flux along the wavy vertical surface follows a power law variation in the vertical direction. Two cases dealing with the Darcy flow model and Forchheimer extended non-Darcy flow model are considered. The non-uniform boundary is transformed to a flat one by a simple transformation obtained using scale analysis and the resulting equations are transformed to boundary layer equations under the assumption that $Ra \gg 1$. The obtained coupled partial differential equations are discretized using the Keller–Box approach [23]. The sparse linear system resulting from the finite difference analysis is solved following the block tri-diagonal solver. The heat transfer and fluid flow process is analyzed using velocity, temperature, streamline and isotherm plots.

2. Mathematical formulation

To study the natural convection process in an isotropic fluid-saturated porous medium let us consider a vertical surface with transverse sinusoidal undulations, immersed in the medium, which is at constant ambient temperature t_∞. It is assumed that the local heat flux rate normal to the surface follows a power law variation, i.e. $q_w = -bx^m$, where x is the vertical cartesian coordinate and b, m are the prescribed constants. The flow model and the coordinate system is depicted in Fig. 1. The origin of the coordinate system is placed at the leading edge of the vertical surface. The surface profile of the vertical wavy wall is given by

$$y = \sigma(x) = \bar{a}\sin\left(\frac{\pi x}{\ell} - \phi\right) \tag{1}$$

To obtain the volume averaged conservation equations the following assumptions are made:

- The flow is steady, laminar, incompressible and two-dimensional.
- The Boussinesq approximation is valid. It neglects all variable property effects in the governing equations, except for the density in the buoyancy term of the momentum equation and approximates the density difference term with a simplified equation of state, i.e. $\rho = \rho_\infty[1 - \beta(t - t_\infty)]$, where β is the volumetric coefficient of thermal expansion.