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On orthogonal polynomial approximation with the dimensional expanding technique for precise time integration in transient analysis ☆

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Abstract

We use four orthogonal polynomial series, Legendre, Chebyshev, Hermite and Laguerre series, to approximate the non-homogeneous term for the precise time integration and incorporate them with the dimensional expanding technique. They are applied to various structures subjected to transient dynamic loading together with Fourier and Taylor approximation proposed in previous works. Numerical examples show that all six methods are efficient and have reasonable precision. In particular, Legendre approximation has much higher precision and better convergence; Chebyshev approximation is also good, but only slightly inferior to Legendre approximation. The other four approximation methods usually produce results with errors hundreds of thousands of times larger. Hermite and Laguerre approximation may be useful for some special non-homogeneous terms, but do not work sufficiently well in our numerical examples. Other contributions of this paper include, a Dynamic Programming scheme for computing series coefficients, a general formula to find the assistant matrix for any polynomial series.

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1. Introduction

In transient analysis, ODEs (Ordinary Differential Equations) in the following form is common and important:

$$\dot{v} = Av + f, \quad v(0) = v_0 \tag{1}$$

where $v(t)$ is an n-dimensional vector function to be determined, A is a given $n \times n$ constant matrix, and $f(t)$ is a given n-dimensional vector function.

For example, the equations of these problems may be transformed into Eq. (1): Dynamic responses of structures subjected to transient loading [1–4], multibody system dynamic models [5], some transient heat transfer problems [6–9] and matrix Riccati differential equations [10,11]. Among such transform, the variational principle is usually used to turn higher order ODEs into first order ones.

In most cases, finding an analytical solution to Eq. (1) is very difficult or even impossible. Thus numerical solution becomes a common alternative. Zhong proposed the Precise Time Integration (PTI) method [3], which can produce numerical results with extremely high precision for Eq. (1) without the non-homogeneous term $f(t)$. Additionally, the PTI was proved to be unconditionally stable and demonstrated adaptivity to stiff problems [1–3].

In order to solve Eq. (1), there are generally two ways: The original one is using the solution theory for ODEs, which first finds one particular solution and then computes the general solution. It requires computing inverse matrices, which is inefficient and induces relatively large errors. Specifically, if the matrix to be inversed is singular or approximately singular, the errors induced are considerable. Therefore the dimensional expanding technique (DET), that transforms non-homogenous ODEs into homogenous ones, is preferable, and adopted by recent works [12] and this paper, which was first proposed by Gu et al. [13].

Function approximation to the non-homogeneous term $f(t)$ is necessary in many cases, because it is usually very difficult or even impossible to find one particular solution if adopting the solution theory for ODEs, and the precondition of the DET i.e. the derivative of the expanded state vector p can be expressed as a linear combination of p itself is also impossible to achieve. Hence efforts have been devoted into this since the very beginning: Originally, Zhong adopted a linear approximation method [3]; Afterwards, Lin et al. applied Fourier approximation [1]; then Zhou et al. carried out Taylor approximation [12].

In this paper, four orthogonal polynomial series, Legendre, Chebyshev, Hermite and Laguerre series, are used to achieve better approximation to $f(t)$ in Eq. (1). The rest of the manuscript is arranged as follows: In Section 2 the PTI for the homogeneous form of Eq. (1) is presented; Sections 3 and 4 discuss the techniques to approximate $f(t)$ and incorporate the DET with the four orthogonal polynomial series respectively; Section 5 is numerical examples where our method is compared with Lin's Fourier approximation [1] and Zhou's Taylor approximation [12]; Section 6 provides some concluding remarks.

2. Precise time integration for the homogeneous form of Eq. (1)

Generally, the homogeneous form of Eq. (1) i.e. with the non-homogenous term $f(t)$ removed should be solved first. Its solution is

$$v = \exp(A \cdot t) \cdot v_0 \tag{2}$$

The time step size is denoted as τ satisfying $t_0 = 0$, $t_1 = \tau, \ldots, t_k = k \cdot \tau, \ldots$ Thus we have the following recursive steps:

$$v_1 = Tv_0, \quad v_2 = Tv_1, \ldots, \quad v_{k+1} = Tv_k, \ldots \tag{3}$$

where $T = \exp(A \cdot \tau)$.

It is seen that, computing the matrix exponential T is the essential, which is illustrated below.

Split the time interval τ into smaller ones. Define $\Delta t = \tau/2^N$, in which N is a natural number. When N is 20, $\Delta t = \tau/1048576$ is very small. Certainly N can be even larger. Then

$$T = [\exp(A \cdot \Delta t)]^{2^N} = (I + T_{a0})^{2^N} = [(I + T_{a0}) \times (I + T_{a0})]^{2^{N-1}} = (I + T_{a1})^{2^{N-1}} \tag{4}$$