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A proper analytical analysis of annular step porous fins for determining maximum heat transfer

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ABSTRACT

The annular stepped fin is a common choice for enhancing heat transfer from a cylindrical primary surface. This typical fin shape is easy in manufacturing compared to the taper shape. There is still effort required to increase the heat transfer rate per unit mass of a step fin. With this design point of view, the present study has been carried out. There may be two design aspects to be adopted to satisfy the above design requirement. With consideration of the porous material and moving condition of fins, heat transfer can be increased for a constraint mass of a fin. In the present study, an appropriate analytical method has been developed for an annular step fin of porous material under a moving condition. The effect of internal heat generation in the fin and the radiation mode of heat transfer on the fin performance have been studied elaborately. Unlike previous literature for moving fins, the performance design parameter is dependent on the modified Peclet number instead of the conventional Peclet number. The double differential transform method has been used for the analytical analysis. The optimization analysis has been demonstrated for the maximization of heat transfer through an annular step porous fin (ASPF) under a design constraint of mass of a fin. A comparative study has also been made between the porous and solid fins for an equal mass of fins. It was highlighted that the porous fin transfer always more heat at an optimum condition compared to the solid fin which is a profitable design aspect for selecting the porous condition. The moving condition is also a better approach to increase heat transfer rate and it is clearly demonstrated in the present study. The formulation of the present work is suitable for stationary and annular disk porous fins also by considering an appropriate design variable.

1. Introduction

Heat transfer by using fins is a classical augmentation technique adopted in many heat exchange applications. There is a continuous effort engaged to modify the fin shape in order to obtain high heat transfer rate per unit mass of fins. With this consideration, different tapered shapes have been suggested by many researchers [1–4]. These tapered shapes save the fin material as well. However, manufacturing these shape is inherent problems as the fin thickness is made of thin for getting high surface area to volume ratio. An alternative shape, namely, step geometry has not only solved the manufacturing problem but also has ability to transfer more heat. The fin material is made of porous to have high heat transfer rate per unit mass of fins.

Hamdan and Al-Nimr [5] used porous fins by enhancing heat transfer in parallel plate channels. The Darcy–Brinkman–Forchheimer model was taken for the flow inside the porous fins. The effective way to enhance heat transfer by using porous media due to mixing of fluid flow and increasing of the contact surface area was investigated by Jiang et al. [6]. Esfahani et al. [7] studied to improve the energy efficiency of heat exchanger by using porous fins. Kiwan and Al-Nimr [8] introduced a novel method for enhancing heat transfer from a given surface by the application of porous fins. The thermal performance of porous fins was compared with that of the conventional solid fins. They found that the porous fin may have an enhancement of performance and saving of 100% of fin material. Saedodin and Sadeghi [9] introduced a simple method of analysis for the performance of porous and solid long fins in natural convection environment. It was studied that the heat transfer rate from porous fin could exceed that of a solid fin. The temperature distribution in porous fins with internal heat generation as a linear function of temperature for Si_3N_4 and AL was predicted by Hatami et al. [10] with three accurate and simple analytical methods, Differential Transformation Method (DTM), Collocation Method (CM) and Least Square Method (LS). Their results reveal that DTM, CM and LS are very effective and accurate in comparison

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Nomenclature

A_j	dimensionless variables defined in Eq. (7) for $j = 1, 2, 3, 4, \ldots, 9$
ASF	annular step fin
ASPF	annular step porous fin
B_j	dimensionless design parameters defined in Eq. (7) for $j = 1, 2, 3, 4$
Bi	Biot number, hr_1/k_s
c_p	specific heat at constant pressure (J kg^{-1} K^{-1})
Da	Darcy number, K/r_1^2
DTM	differential transform method
$f(\psi, R_1)$ $g(\psi, R_1)$	optimization functions defined in Eqs. (17a) and (17b), respectively
$F(j), G(j)$	differential transform functions
g	gravitational acceleration (m s^{-2})
Gr	Grashof number, $g\beta'(T_b - T_\infty)r_1^3/\nu_f^2$
h	convective heat-transfer coefficient (W m^{-2} K^{-1})
k	thermal conductivity (W m^{-1} K^{-1})
K	permeability of the porous fin (m^2)
k_r	thermal conductivity ratio, k_f/k_s
$\dot{m}$	mass flow rate of fluid per unit area (kg s^{-1} m^{-2})
Pe	Peclet number, $Re\ Pr$
Pe^*	modified Peclet number, $Pe\alpha_f/\alpha_s$
Pr	Prandtl number, ν_f/α_f
q	conduction flux, see Eq. (1) (W m^{-2})
q_a	actual heat-transfer rate through a fin (W)
q_g	dimensionless internal heat generation, $q_0''' r_1^2/T_b k_s$
q_s	heat flux through the outer surface of the fin, see Eq. (3) (W m^{-2})
q'''	volumetric heat generation rate as a linear function of temperature (W m^{-3})
q_0'''	volumetric heat generation rate corresponding to T_∞
q_i	ideal heat-transfer rate through a fin (W)
Q	dimensionless actual heat-transfer rate, $q_a/4\pi r_1 k_s T_b$
Q_i	dimensionless ideal heat-transfer rate, $q_i/4\pi r_1 k_s T_b$
Q_{opt}	optimum or maximum value of Q
r	radial coordinate starting from the center of the tube (m)
r_1	inner radius (m)
r_2	step radius of an ASF (m)
r_3	outer radius (m)
R	dimensionless radial coordinate, r/r_3
R_1	dimensionless inner radius, r_1/r_3
R_2	dimensionless step radius, r_2/r_3
Ra	Rayleigh number, $Gr\ Pr$
Ra^*	modified Rayleigh number, $Ra\ Da/2$
Re	Reynolds number, $\upsilon_r r_1/\nu_f$
R_p	dimensionless surface radiation-convection parameter, $\sigma\varepsilon T_b^3/h$
R_t	dimensionless radiative-conductive parameter, $4\sigma T_\infty^3/3\beta_R k_s$
t_1	semi-base thickness (m)
t_2	semi-tip thickness (m)
T	local fin surface temperature (K)
T_∞	surrounding temperature (K)
T_b	base temperature (K)
T_i	fin surface temperature at the step section (K)
$T_{\max}$	maximum fin surface temperature in the fin (K)
U	dimensionless fin volume, $V/2\pi r_1^3$
U^*	dimensionless mass of a fin per unit density of the solid, $(1-\phi)U$
υ_r	radial velocity of solid (m s^{-1})
V	fin volume (m^3)
X	dimensionless coordinate, x/r_3
x	coordinate starting from the tip or from the step change section towards the center of the tube (m)
$y(r)$	an arbitrary function
$Y(j)$	transform function of $y(r)$
Z_0	dimensionless fin parameter, $\sqrt{Bi/\psi}$

Greek letters

α	thermal diffusivity (m^2 s^{-1})
β'	linear variable heat-generation parameter (K^{-1})
β	dimensionless variable heat-generation parameter, $\beta'(T_b - T_a)$
β_R	Rosseland extinction coefficient
ε	emissivity of fin material
δ	kronecker delta
η	fin efficiency
σ	Boltzmann constant (W m^{-2} K^{-4})
ψ	dimensionless thickness, t/r_1
τ	thickness ratio, t_2/t_1
θ	dimensionless temperature, T/T_b
θ_a	dimensionless surrounding temperature, T_∞/T_b
θ_i	dimensionless temperature at the interface, T_i/T_b
$\theta_{\max}$	dimensionless maximum temperature, $T_{\max}/T_b$
ϕ	porosity
ρ	density (kg m^{-3})
ν	kinematic viscosity (m^2 s^{-1})

Subscripts

f	fluid
s	solid

to the numerical results. Cuce and Cuce [11] applied successfully of Homotopy perturbation method for efficiency and effectiveness assessment of longitudinal straight porous fins. They also determined the ratio of porous fin to solid fin heat transfer rate as a function of thermo-geometric fin parameters.

The above literature survey for porous fins [5–11] has been carried out for the constant thickness fins. As the heat conduction decreases from the fin base to fin tip the shape of the fin should be convergent in the direction of heat conduction for effective utilization of fin material. The requirement of solid material declined with the porosity for transferring same heat has been determined by Kundu and Lee [12] by an exact analysis for minimum shape of porous fins. Vahabzadeh et al. [13] analytically investigated the porous pin fins with rectangular, triangular, convex parabolic and concave parabolic profiles in fully wet conditions. Straight porous fins of four different profiles, namely, rectangular, convex parabolic and two exponential types were considered by Kundu et al. [14] to establish a performance and optimum design analysis analytically. Hatami and Ganji [15] determined heat transfer and temperature distribution for circular convective–radiative porous fins for four different shapes, rectangular, convex, triangular and exponential geometries. Moradi et al. [16] studied the convection and radiation effects on the performance of a porous triangular fin with temperature-dependent thermal conductivity. For the semi-spherical porous fin, Hatami et al. [17] established a refrigeration efficiency analysis for fully wet conditions.

Many processes in industries such as extrusion, hot rolling, glass, fibber drawing, and casting, models as moving fin, where the heat transfer from the extruded products, and rolled sheet to the surroundings takes place in continuous motion [18]. The fins