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Forward and inverse solutions of a conductive, convective and radiative cylindrical porous fin

Ranjan Das

School of Mechanical, Materials and Energy Engineering, Indian Institute of Technology, Rupnagar, Nangal Road, Punjab 140001, India

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ABSTRACT

This paper deals with the numerical study of a conductive, convective and radiative cylindrical porous fin. At first, Runge–Kutta method-based numerical solution is obtained for calculating the temperature distribution, and then an inverse problem is solved for estimation of unknown parameters. Five critical parameters such as the porosity, emissivity, solid thermal conductivity, thickness and the permeability have been simultaneously predicted for satisfying a prescribed temperature distribution on the surface of the porous fin. This is achieved by solving an inverse problem using the hybrid evolutionary–nonlinear programming optimization algorithm. The effect of random measurement errors between ±10% has been considered. The estimated values of non-dimensional entities such as porosity and surface emissivity are found to be approximately within the range, 0.28–0.92 and 0.27–0.75, respectively. Additionally, the thermal conductivity, thickness and the permeability are found to be almost between 17 and 140 W/m K, 8.7×10^{-4} to 0.007 m and 2×10^{-11} to 5×10^{-8} m^2, respectively. The present study reveals that many feasible combinations of available materials satisfy the same temperature field, thus providing an opportunity for selecting any combination from the available alternatives. Moreover, the hybrid method is found to perform better and yield relatively faster convergence than individual methods. The sensitivity analysis reveals that the effect of fin permeability on the temperature field is considerably high than other parameters.

1. Introduction

The heat transfer analysis of porous fins is one of the emerging fields of research, since, for the same weight, a porous fin performs better than a solid fin [1]. Apart from the initial and boundary conditions, the temperature distribution in a porous fin depends upon many thermo-physical properties such as density, specific heat, thermal conductivity, dynamic viscosity and coefficient of volumetric thermal expansion of the fluid. In addition to these properties, solid thermal conductivity, permeability, porosity along with the surface conditions and geometric configurations also influence the temperature field of a porous fin. The available literatures indicate that most of the studies on porous fins are aimed at calculating the steady-state temperature distributions from the knowledge of thermo-physical properties using appropriate boundary conditions of the fin [2–8]. One of the ways of obtaining the temperature profile is to conduct an experiment by maintaining proper boundary conditions. However, it is well-known that for saving the time, money and manpower, a computational study is preferable. In order to computationally achieve the desired task, the governing heat transfer equation needs to be solved by employing a suitable method using known thermo-physical properties and boundary conditions. Such type of approach is known as forward analysis [9,10] and the solution of such analyses is generally unique. The situation becomes relatively different and complex when the objective is to determine some important unknown parameters in order to satisfy a particular temperature field. Such type of problems is known as inverse problems and their solution is not necessarily unique [11]. In other words, several possible combinations of unknown parameters satisfying a given requirement may exist [12]. Therefore, the investigation of inverse problems is an important and pertinent task for designing an engineering system [13].

It is observed on one hand that many studies dealing with inverse analysis of conventional fins are available in the literature [14]. However, on the other hand very few inverse problems for porous fins are available [12,15]. The solution of an inverse problem requires a forward method along with a suitable optimization method, and its solution depends upon the effectiveness of the optimization algorithm. Many optimization methods can be found in the literatures which have been employed for solving inverse heat transfer problems involving fins. Some of them are the

E-mail address: ranjandas81@gmail.com

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Nomenclature

A	matrix of constraint gradients
a, b, c, d, e	vectors in DE algorithm
c_p	specific heat of the fluid
Da	Darcy number
e_r	measurement error
F	objective function
f	radiation shape factor
g	acceleration due to gravity (9.81 m/s^2)
Gr	Grashof number
H	Hessian matrix in NLP algorithm
j	set containing the constraints
K	permeability of the fin (m^2)
k	thermal conductivity (W/m K)
k_f	fluid thermal conductivity (W/m K)
k_r	thermal conductivity ratio, k_{eff}/k_f
k_s	solid thermal conductivity (W/m K)
$\dot{m}$	mass flow rate of the fluid (kg/s)
N	number of temperature measurement points
N_c	convection parameter
N_r	radiation parameter
N	set containing the unknowns
P	matrix of slack variables
p	number of constraints
Pr	Prandtl number
q	heat transfer rate (W)
R	any location along the radial direction (m)
r_b	inner (base) radius of the fin (m)
r_t	outer (tip) radius of the fin (m)
r^*	non-dimensional radius (=r/r_b)
r_t^*	non-dimensional tip radius (=r_t/r_b)
Ra	Rayleigh number (=Gr·Pr)
S	sensitivity coefficient
s	slack variable
T	temperature (K)
T_b	base temperature (K)
T_∞	ambient temperature (K)
t	thickness of the fin (m)
Y	mutant vector in DE algorithm
Z	parent vector in DE algorithm
z	child vector in DE algorithm

Greek symbols

α_f	thermal diffusivity of the fluid (m^2/s) (=$k_f/(\rho \cdot c_p)$)
β	coefficient of volumetric thermal expansion (1/K)
χ	crossover probability in DE algorithm
ε	emissivity of the fin surface
ϕ	porosity of the fin
μ	penalty parameter in NLP
ν_f	kinematic viscosity of the fluid (m^2/s)
θ	non-dimensional temperature (=T/T_b)
θ_a	non-dimensional ambient temperature (=T_∞/T_b)
$\tilde{\theta}$	exact value of non-dimensional temperature
ρ	density of the fluid (kg/m^3)
σ	Stefan–Boltzmann constant (5.67×10^{-8} W/m^2 K^4)
ω	scaling factor in DE algorithm
ψ	Lagrange multiplier in NLP algorithm

conjugate gradient method [16], Levenberg–Marquardt method [17], genetic algorithm (GA) [18], simplex search method [19], simulated annealing (SA) [20], differential evolution (DE) [21], etc. Apart from to these, for problems other than inverse analyses of fins, many optimization methods working on nonlinear programming (NLP) methods are also well-addressed in the literature [22,23]. Inspite of relatively slow converging feature of the evolutionary/stochastic methods than the deterministic methods due to elimination of gradients [24], it is observed that for inverse problems, the stochastic/evolutionary-based optimization algorithms (such as GA, DE, and SA) work better than the conventional deterministic methods [25]. This is due to the reason that unlike deterministic algorithms which work either with one or few solutions at a time, the stochastic/evolutionary algorithms possess the capability of working simultaneously with more number of feasible solutions. Therefore, the stochastic/evolutionary algorithms can perform exhaustive searching in the solution search domain. As compared to other evolutionary optimization algorithms, DE [26] possesses few advantages such as fast convergence, simplicity, easy of implementation and its efficacy for optimizing many engineering and nonlinear problems [27,28]. For inverse problems, in order to combine the advantages of deterministic methods (such as fast convergence) and stochastic methods (such as the capability of exhaustive searching) into one algorithm, recently the usage of hybrid optimization techniques are also gaining considerable attention [29].

Due to the inherent advantages over conventional fins and very limited availability inverse optimization studies for porous fins, the objective of the present work is to provide forward numerical and inverse solutions for a porous cylindrical fin. At first, using fourth order Runge–Kutta (RK-4) method, numerical solutions for computing the temperature field is obtained. The temperature field of the present work is then validated with the results of the homotopy analysis method (HAM) described in the relevant literature [30]. During the inverse analysis, a hybrid DE–NLP algorithm is used for the purpose of optimization due to various reasons as mentioned earlier. The inverse problem is aimed at simultaneously estimating five critical parameters such as the fin porosity, emissivity, solid thermal conductivity, thickness and permeability of the cylindrical porous fin for satisfying a prescribed temperature field. In this work, as the temperature field is assumed to be available only for a particular radial distance, so the inner and the outer radii are assumed to remain fixed. Since there are five unknowns, so it is most likely that many feasible combinations of the five unknowns can satisfy a given requirement, and this will offer flexibility in selecting any combination of the five parameters amongst many feasible alternatives. The solid material for the porous fin can be selected from the known value of the solid thermal conductivity and therefore solid thermal conductivity is treated as an unknown quantity for the inverse problem. The fluid properties are assumed to remain fixed and nearly correspond to air at 300 K and 1 atmosphere pressure. For a given temperature field, the unknown properties such as the fin porosity and thickness can be adjusted by the designer, therefore these are treated as unknown parameters. In addition to these parameters, the emissivity can be regulated by adjustment of surface conditions as the emissivity increases with an increase in the surface roughness [31–33]. Additionally, the permeability (which is the measure of the ease with which the fluids can pass through the solid) of a porous material depends upon the size of the pores, pressure regulation, surface conditions, etc. [34,35]. Due to these reasons, the emissivity and permeability are also considered to be unknown parameters during the inverse analysis. Below the formulation and solution methodology of the present problem are discussed.

2. Formulation

Let us consider the geometry of a porous concentric cylindrical fin as shown in Fig. 1. The analysis is subjected to the following assumptions [3,5,8,10,12,30,36–42].