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Effect of the ultrasonic emitter on the distortion performance of the parametric array loudspeaker

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A R T I C L E I N F O

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A B S T R A C T

The parametric array loudspeaker (PAL) is a type of directional loudspeaker that utilizes the nonlinear acoustic effect to create the audible sound in an ultrasonic beam. Due to this unusual sound principle, it is inevitable that nonlinear distortion is incurred in the sound transmission of the PAL. Numerous modulation methods aiming to reduce the nonlinear distortion have been developed on the basis of the Berktay's far-field solution, but they often perform in an unexpected manner. The degraded practical performance has been credited to the inaccuracy of the Berktay's far-field solution. In this paper, we demonstrate the effect of the ultrasonic emitter on the distortion performance of the PAL and suggest that the Berktay's far-field solution remains to be a good model equation.

1. Introduction

The PAL is able to transmit a narrow sound beam in air from a small sized ultrasonic emitter [1,2]. This ability is resultant from the nonlinear acoustic effect of an ultrasonic beam that consists of two frequencies, whereby the difference frequency is generated as one of the extraneous frequencies [3]. The directivity of the difference frequency is described by an end-fire array, which gives a similarly narrow beamwidth as the ultrasonic beam [4,5]. The PAL is readily adopted in various sound applications, such as active noise control [6], audio projection [7], human–machine interface [8], and even in an increasing number of contemporary art works [9,10].

Fig. 1 shows the block diagram of the PAL. The audio input is modulated on an ultrasonic carrier. The modulated input becomes an ultrasonic signal, which is then amplified to drive the ultrasonic emitter. The nonlinear acoustic effect in air distorts the waveform transmitted from the ultrasonic emitter and thus creates the audible sound. It is noteworthy that as compared to the ultrasound level, the audible sound pressure level is relatively weak. The ultrasound level must be controlled under safety regulations [11].

The Khokhlov–Zabolotskaya–Kuznetsov (KZK) equation describes the combined nonlinear acoustic effect of absorption, diffraction, and nonlinearity. It is one of the most efficient model equations of the PAL, but there is no analytical solution to the KZK equation [12]. Alternatively, Berktay provided a simple model equation for the far-field on-axis case, which states that the audible sound pressure level is proportional to the second derivative of the squared envelope of the modulated input [13].

Modulation methods have been developed for the PAL. The double sideband (DSB) modulation method was carried out in the world first PAL [1]. Based on the Berktay's far-field solution, the second harmonic distortion level of the DSB modulation method is proportional to the modulation index and the frequency response possesses a slope of 12 dB per octave as a result of the second derivative. Kamakura et al. [14] proposed the square root (SRT) modulation method that provided an inverse system to the Berktay's far-field solution. Further development of the SRT modulation method preprocessed the audio input by a double integral to offset the 12 dB per octave slope [15].

There is another trend of the modulation method. The single sideband (SSB) modulation method, which is a type of quadrature modulation method, has been studied since the early days of the PAL [16]. The SSB modulation method includes a quadrature path to cancel the nonlinear distortion. The DSB and SSB modulation methods have individual advantages. So far, there have been two hybrid modulation methods combining the DSB and SSB modulation methods. The weighted DSB modulation method makes use of the relative high audible sound pressure level of the DSB modulation method to enhance the SSB modulation method at the low frequency band [17], while the asymmetrical amplitude modulation (AAM) method is a part of the audio bandwidth extension technique for the PAL [18,19].

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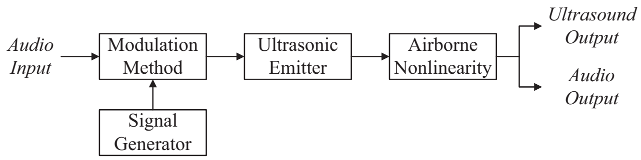

Fig. 1. Block diagram of the PAL.

Similar to the SSB modulation method, the modified amplitude modulation (MAM) methods are quadrature modulation methods too [20]. The quadrature path in the SSB modulation method is often implemented by the Hilbert filter, while quadrature paths in the MAM methods are calculated by polynomial equations. The MAM methods using higher order polynomial equations are developed to achieve better distortion performance. Previous studies of the MAM methods provide simulation results only [21]. There is a lack of experiment validation of the MAM methods [22,23].

Therefore, this paper aims to highlight the effect of the ultrasonic emitter on the distortion performance of the PAL. Modulation methods are comparatively evaluated under the same condition by simulations and measurements. The Berktay's far-field solution and the Merklinger's far-field solution are adopted as the model equations in the simulation [13,24]. It is found that the discrepancy between the theoretical and measurement results is greater for the more sophisticated modulation method, if the frequency response of the ultrasonic emitter is ignored.

2. Model equations

2.1. Berktay's far-field solution

To derive the Berktay's far-field solution, the primary sound pressure level is assumed in the one-dimension form of

$$p_1 = P_0 E(t)\exp(-\alpha_0 z)\cos(\omega_0 t), \tag{1}$$

where P_0, α_0, and ω_0 are initial pressure level, attenuation rate, angular frequency of the ultrasonic carrier, respectively; t is the retarded time; z is the on-axis coordinate; and $E(t)$ is the envelope of the modulated input, which is assumed to have an unit amplitude.

Subsequently, the audible sound source strength density is written as

$$q_d = \frac{\beta P_0^2}{\rho_0^2 c_0^4}\exp(-2\alpha_0 z)\frac{\partial}{\partial t}\left[\frac{E^2(t)}{2}\right], \tag{2}$$

where β is the nonlinear coefficient; ρ_0 is the density of air; and c_0 is the speed of sound in air.

The audible sound pressure level is thus calculated by

$$p_d(x) = \frac{\rho_0 a^2}{4x}\int_0^{+\infty}\frac{\partial q_d}{\partial t}dz = \frac{\beta P_0^2 a^2}{16\rho_0 c_0^4\alpha_0 x}\frac{\partial^2}{\partial t^2}E^2(t), \tag{3}$$

where x is the observation point and a is the radius of the ultrasonic emitter.

2.2. Merklinger's far-field solution

In Berktay's derivation, the primary sound pressure level is assumed to decrease with distance by the thermoviscous absorption effect only. Merklinger extended the Berktay's far-field solution to include the nonlinear absorption effect [25]. The energy transferred from the primary sound to the second harmonic of the primary sound, which is much greater than the energy transferred from the primary sound to the audible sound, attenuates the primary sound pressure level to become

$$p_1 = \frac{P_0 E(t)\exp(-\alpha_0 z)}{\sqrt{1+\Gamma^2 E^2(t)[1-\exp(-2\alpha_0 z)]^2/16}}\cos(\omega_0 t), \tag{4}$$

where Γ is called the Gol'dberg number [26] and defined as

$$\Gamma = \frac{\beta P_0 \omega_0}{\rho_0 c_0^3 \alpha_0}. \tag{5}$$

If a modified envelope function is correspondingly defined as

$$E'(t) = \frac{E(t)}{\sqrt{1+\Gamma^2 E^2(t)[1-\exp(-2\alpha_0 z)]^2/16}}, \tag{6}$$

(2) and the integral in (3) are still valid to calculate the audible sound pressure level. Therefore, the Merklinger's far-field solution is given by

$$p_d(x) = \frac{P_0 a^2}{4\omega_0 c_0 x}\frac{\partial^2}{\partial t^2}\{E(t)\tan^{-1}[\Gamma E(t)/4]\}. \tag{7}$$

When $\Gamma E(t)/4$ is small, substituting $\tan^{-1}[\Gamma E(t)/4] \approx \Gamma E(t)/4$ into (7) yields the Berktay's far-field solution. When $\Gamma E(t)/4$ is large, $\tan^{-1}[\Gamma E(t)/4]$ is approximated by $\mathrm{sgn}[E(t)]\pi/2$. This makes (7) simplified to a concise expression as

$$p_d(x) = \frac{P_0 \pi a^2}{8\omega_0 c_0 x}\frac{\partial^2}{\partial t^2}|E(t)|. \tag{8}$$

Most of the PALs utilize the ultrasonic carrier at 40 kHz. When the ultrasonic carrier is transmitted at the initial pressure level of 110 dB, the Gol'dberg number in the general room condition approximates 0.4 [26]. Therefore, the Berktay's far-field solution is valid and the second order nonlinearity $E^2(t)$ determines the audible sound pressure level. However, when the initial pressure level increases to 130 dB, the Gol'dberg number is proportional to the initial pressure level and becomes 4.0. The audible sound pressure level is associated with $E(t)\tan^{-1}E(t)$, as described by the Merklinger's far-field solution. Since the safety of using ultrasound in public is an important concern, the initial pressure level of the ultrasonic carrier is more likely to be set close to 110 dB rather than 130 dB.

3. Modulation methods

3.1. DSB modulation method

The DSB modulation method is the first modulation method applied in the PAL [1]. The envelope of the DSB modulation method is written as

$$E^{DSB}(t) = 1 + mx(t), \tag{9}$$

where m is the modulation index and $x(t)$ is the audio input. The disadvantage of the DSB modulation method has been well known. Substituting (9) into the Berktay's far-field solution yields the audible sound pressure level as

$$p_d^{DSB} \propto \frac{\partial^2}{\partial t^2}\left[2mx(t) + m^2x^2(t)\right]. \tag{10}$$

The second harmonic distortion level of the DSB modulation method is credited to the second term in the bracket.

The distortion performance of the DSB modulation method is improved when the Gol'dberg number is increased. If the Merklinger's far-field solution becomes the dominating model equation, substituting (9) into (8) yields the audible sound pressure level as