



# Theoretical analysis of plate vibration due to acoustic signals



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## ABSTRACT

Acoustic emission is high frequency elastic waves produced in a plate under load by the rapid release of strain energy during crack propagation. These waves propagated through the plate material. Knowledge of how the plate is vibrated by acoustic emission resulting from crack growth is an important parameter for crack locating. In this paper vibration behavior of Al–Mg plate due to acoustic signals is modeled. The displacement field was used to derive the governing motion equations of the first-order shear deformation theory for plate by means of Hamilton's principle. The displacement field was written based as double Fourier series. A MATLAB code according Runge–Kutta numerical method was generated to solve the motion equations of plate. To check the theoretical results, experiments were carried out with four sensors installed on a plate of 5056 Al–Mg with dimensions of 1 m \* 1 m in a rectangular array. An impact was generated by an artificial AE source such as Hsu–Nielsen method of pencil lead breaking (PLB) at any position of the plate. Resonance frequencies of the used AE sensors are employed to investigate the vibration behavior of plate due to acoustic emission. In the next step, Fast Fourier Transform was taken from theoretical and experimental results. Comparing the obtained results showed that the frequency which carried the most amount of energy is approximately the same which confirm performance of the developed analytical model.

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## 1. Introduction

When a material structure is distorted or damaged by an internal or external force or micro structural changes occurred in particle, it will release elastic waves propagated through the material. This phenomenon named as acoustic emission (AE) is a robust and popular technique to evaluate and investigate damage propagation in the structures. Crack, friction and corrosion are some micro structural changes mentioned above [1]. Source localization has always been regarded as one of the main applications of AE testing which is accomplished based on the wave velocity and arrival time differences at the limited number of sensors. Different approaches have been developed to locate AE sources. The traditional method for this procedure is determining arrival times using threshold, which means the time when a signal crosses the threshold for the first time [2]. Ziola and Gorman [3] discussed about substantial errors of source localization results by this procedure. More accurate methods have also been used to consider actual time of arrival (TOA) at the different sensors. A hyperbola obtained using different arrival times and wave velocity, represents possible

locations of the source for each pair of the sensors. The location of the source on the plate is determined at the intersection points of two hyperbolas [4]. Some researchers have investigated generic methods for source locating on anisotropic materials. Scholey et al. [5] published a method where a map was generated on the plate with arrival time differences ( $\Delta t$ ) for each pair of sensors.  $\Delta t$  was calculated using sensor distances and wave velocity dependency on the fiber orientation. This technique is named as “best match point search method”. An attempt to provide a better theoretical background for AE testing is modal acoustic emission (MAE) [6]. Elastic waves based on their mechanical behavior can propagate through the materials at the different modes named as extensional and flexural. The separation of these modes could make it possible to elicit exact information about the source of elastic waves. This is an appropriate technique for source locating using one sensor. Surgeon and Wevers [7] applied MAE technique in composite laminates to analyze AE signals. Ding et al. [8] developed a new waveform analysis to estimate AE wave arrival times using wavelet transform. Varieties of techniques were compared in proposed method such as threshold crossing, cross correlation and wavelet packet transform. A method based on wavelet decomposition is recommended as the most consistent and accurate method for determining arrival time. Mostafapour and Davoodi [9] presented a method based on wavelet transform, filtering and

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cross correlation techniques to detect and locate continuous leakage source in underground high pressure gas pipe. They carried out the experiments to survey the accuracy of their method and the maximum error percent of 5% was achieved for leak locating. Acoustic emission resulting from crack growth transmitted through the plate material can be recorded by accelerometers. Knowledge of how these waves can vibrate the plate and the vibration behaviors of plate are very important in crack locating process. Chi and Chung [10] studied the functionally graded elastic rectangular plate under transverse loading. The Poisson's ratio of the functionality graded plate was assumed constant but their Young's module and densities was varied continuously throughout the thickness direction according to volume fraction function of components. Kandasamy and Singh [11] presented a numerical method based on the Rayleigh–Ritz method for the forced vibration of open cylindrical shell. The motion equations were derived from the three-dimensional strain–displacement relations in the cylindrical coordinate system. The transient response of the shell with and without damping was sought by transforming the equation of motion to the state-space model and using the Runge–Kutta algorithm. Ran [12] surveyed an analytical solution for the vibration response of a ribbed plate clamped on all its boundary edges by employing a travelling wave solution. The dynamic characteristics and mode shapes of the ribbed plate were measured and compared to those obtained from the analytical solution and finite element analysis. Zhao et al. [13] investigated the vibration and acoustic response characteristics of orthotropic laminated composite plate with simply supported boundary conditions excited by a harmonic concentrated force. To verify the theoretical solution, numerical simulations were carried out. Their results showed that the dynamic and acoustic response reduced and the coincidence frequency decreased with the enhanced stiffness. Mostafapour and Davoudi [14] employed Donnell's non-linear theory for modeling pipe vibration due to leakage and derivation of motion equation of pipe in radial direction. They carried out the experiments with continuous leak source. Comparing the Fast Fourier Transform results showed good agreement between the theoretical and experimental results.

In this study, Al–Mg plate vibration behavior due to an artificial AE source is modeled. AE waves produced by pencil lead breaking, propagated through the plate and caused vibration in all direction. This source, propagated waves in a large of frequencies about 0–400 kHz. The first shear deformation theory is used to derive the motion equations of plate in all directions. To solve these equations, a MATLAB code based on the Runge–Kutta method is employed and the plate displacements are obtained. To survey the accuracy of the analytical results, acoustic emission tests were carried out with AE source of Hsu–Nielsen.

## 2. Motion equation for plate

A rectangular plate with coordinate system ( $O; x, y, z$ ), having the origin  $O$  at one corner is considered as shown in Fig. 1. The displacements of an arbitrary point of coordinates  $(x, y)$  on the middle surface of the plate are denoted by  $u, v, w$  in the  $x, y$  and out-of plane ( $z$ ). The dimensions of plate in  $x$  and  $y$  directions are  $a$  and  $b$  respectively and the plate thickness is  $h$ . In this study, the equation of motion for a double curved shell was derived in general form and then, with some simplifications, the equations for flat plates were obtained. Based on a moderately thick shell theory [15], the following forms of the displacement fields are assumed:

$$\begin{aligned} u(x, y, z, t) &= u_0(x, y, t) + z\phi_1(x, y, t) \\ v(x, y, z, t) &= v_0(x, y, t) + z\phi_2(x, y, t) \\ w(x, y, z, t) &= w_0(x, y, t) \end{aligned} \quad (1)$$

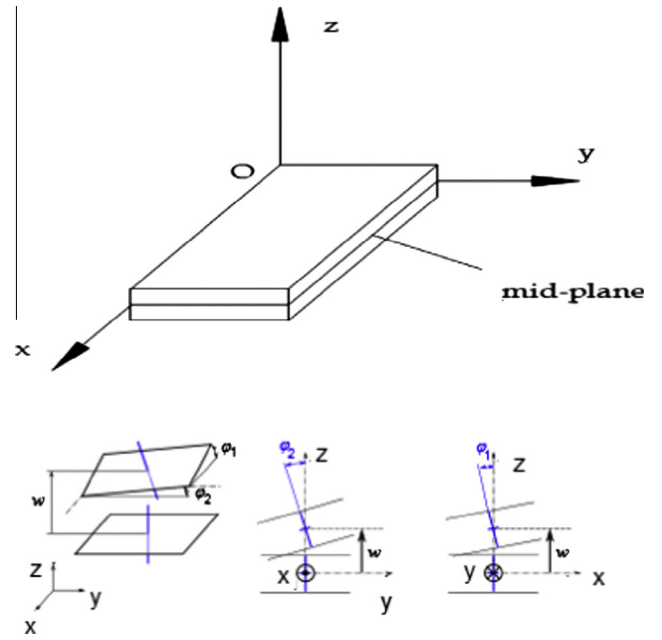


Fig. 1. Plate and coordinate system.

in which,  $x, y, z$  are orthogonal curvilinear coordinate system,  $(u_0, v_0, w_0)$  are the displacement of a point  $(x, y, 0)$  on the mid-surface of the shell and  $(\phi_1, \phi_2)$  are the rotations of a normal to the reference surface (Fig. 1). Strain–displacement relation for a double curved shell is obtained as follow [16]:

$$\begin{aligned} \epsilon_{11} &= \frac{1}{(1+z/R_1)} (\epsilon_{11}^0 + zk_{11}), & \epsilon_{22} &= \frac{1}{(1+z/R_2)} (\epsilon_{22}^0 + zk_{22}), & \epsilon_{33} &= 0 \\ \gamma_{12} &= 2\epsilon_{12} = \frac{1}{(1+z/R_1)} (\epsilon_{12}^0 + zk_{12}) + \frac{1}{(1+z/R_2)} (\epsilon_{21}^0 + zk_{21}), \\ \gamma_{13} &= 2\epsilon_{13} = \frac{1}{(1+z/R_1)} \epsilon_{13}^0 & \gamma_{23} &= 2\epsilon_{23} = \frac{1}{(1+z/R_2)} \epsilon_{23}^0 \end{aligned} \quad (2)$$

where:

$$\begin{aligned} \epsilon_{11}^0 &= \frac{\partial u_0}{\partial x} + \frac{w_0}{R_1} & k_{11} &= \frac{\partial \phi_1}{\partial x} & \epsilon_{22}^0 &= \frac{\partial v_0}{\partial y} + \frac{w_0}{R_2} \\ k_{22} &= \frac{\partial \phi_2}{\partial y} & \epsilon_{12}^0 &= \frac{\partial v_0}{\partial x} + \frac{\partial u_0}{\partial y} \\ k_{12} &= \frac{\partial \phi_2}{\partial x} + \frac{\partial \phi_1}{\partial y} - C_0 \left( \frac{\partial v_0}{\partial x} - \frac{\partial u_0}{\partial y} \right) & \epsilon_{23}^0 &= \frac{\partial w_0}{\partial y} + \phi_2 - \frac{v_0}{R_2} \\ \epsilon_{13}^0 &= \frac{\partial w_0}{\partial x} + \phi_1 - \frac{u_0}{R_1} \\ C_0 &= \frac{1}{2} \left( \frac{1}{R_1} - \frac{1}{R_2} \right) \end{aligned} \quad (3)$$

in which,  $R_1$  and  $R_2$  are the values of the principal radii of curvature of the middle surface and  $C_0$  is considered in order to prevent of the rigid body rotation. The displacement field (Eq. (1)) can be used to derive the governing equations of the first-order shear deformation theory of shells by means of Hamilton's principle. Therefore:

$$\int_0^T \delta L dt = \int_0^T [\delta K - (\delta U + \delta U_i + \delta U_s) + \delta W_{\text{ext}}] dt = 0 \quad (4)$$

in which,  $L$  is differential operator,  $U, K, W_{\text{ext}}$  are kinetic, strain and external works energy respectively. The external loads applied to the plate caused by AE source is considered as follow:

$$q(x, y, t) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} q_{(m,n)}(t) e^{i\omega t} \text{Sin}(\alpha_m x) \text{Sin}(\beta_n y) \quad (5)$$

where  $\alpha_m = \frac{m\pi}{a}$ ,  $\beta_n = \frac{n\pi}{b}$  and  $\omega$  is the frequency of applied force and  $m, n$  are the wave number along  $x, y$  directions, respectively.

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