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Three-dimensional coupled vibration theory for the longitudinally polarized piezoelectric ceramic tube

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ABSTRACT

Longitudinally polarized piezoelectric ceramic devices have been widely used in underwater sound and ultrasonic transducers. When the geometrical dimensions satisfy certain conditions, the vibration can be regarded as one-dimensional vibration, such as the longitudinal vibration of a slender ceramic rod, or the plane radial vibration of a thin piezoelectric ring. However, for the longitudinally polarized piezoelectric ceramic tube with comparable longitudinal and radial geometrical dimensions, because of its complexity, its coupled vibration has not been studied up to now. In this paper, based on three-dimensional (3D) motion equations and electrostatic charge equation, according to the free boundary conditions of the piezoelectric tube, the analytical resonance frequency equation of longitudinal–radial coupled vibration for the longitudinally polarized piezoelectric tube is obtained. Its vibrational distribution is simulated by using ANSYS software. The results show that the analytical resonance frequencies are in good agreement with the numerical results and the piezoelectric tube vibrates in radial and longitudinal directions. So, it is expected that this theory can serve as reference for the design of the longitudinal–radial composite vibrational systems.

1. Introduction

Many electroacoustic devices used nowadays in various fields [1–5] are based on piezoelectric phenomenon, which concerts electric signals into acoustic signals and vice versa [6]. The key elements of them are piezoelectric vibrators with different shapes, such as circular disk, ring, tube, cylinder and rectangular plate. These piezoelectric ceramic devices can be excited to vibrate in different vibrational modes, determined by the geometrical shape, the dimensions, the frequency range, the polarization direction and the direction of external exciting electric field.

The radially polarized piezoelectric ceramic thin-walled rings, tubes, and cylindrical transducers were studied in previous literatures [7–9]. On the other hand, Mason electro-mechanical equivalent circuits for longitudinally polarized piezoelectric ceramic rings, disks and thin-walled cylinders have also been obtained [10–12]. And these rings or disks have also been widely used in longitudinal or radial ultrasonic and underwater sound transducers [13–15]. In these studies, the piezoelectric devices are assumed to be isotropic, or thin-walled, and the shearing stress and radial normal stress are generally ignored. However, for the longitudinally polarized piezoelectric ceramic tube with comparable longitudinal and radial geometrical dimensions, because of the complexity, its coupled vibration has not been studied up to now [16–18].

In the present paper, the longitudinally polarized piezoelectric ceramic tube with comparable longitudinal and radial geometrical dimensions is studied. Based on three-dimensional motion equations and electrostatic charge equation, resonance frequency equation of its longitudinal–radial coupled vibration is obtained analytically. Then, its vibration distribution was simulated by using ANSYS software. It is expected that this theory can serve as reference for the design of the longitudinal–radial composite vibrational systems.

2. Three-dimensional analysis of coupled vibration for longitudinally polarized piezoelectric ceramic tube

The longitudinally polarized piezoelectric ceramic tube with comparable longitudinal and radial geometrical dimensions is schematically shown in Fig. 1. Its outer and inner radii are a and b, respectively. Its length l is comparable with the outer radius a. The polarization direction is along the height of the tube and external exciting electric field is parallel to the polarization direction. In cylindrical coordinates (r, θ, z), Fig. 1 is an axial-symmetric tube, so,

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its stress and strain can be expressed as four independent variables $S_r, S_\theta, S_z, S_{rz}$ ($S_{r\theta} = S_{\theta z} = 0$) and $T_r, T_\theta, T_z, T_{rz}$ ($T_{r\theta} = T_{\theta z} = 0$), respectively [19]. The three-dimensional motion equations and electrostatic charge equation for the tube in longitudinal–radial coupled vibration are as follows:

$$\begin{cases} \rho \frac{\partial^2 \xi_r}{\partial t^2} = \frac{\partial T_r}{\partial r} + \frac{\partial T_{rz}}{\partial z} + \frac{T_r - T_\theta}{r}, \\ \rho \frac{\partial^2 \xi_z}{\partial t^2} = \frac{\partial T_{rz}}{\partial r} + \frac{\partial T_z}{\partial z} + \frac{T_{rz}}{r}, \\ \frac{\partial D_r}{\partial r} + \frac{1}{r} \cdot \frac{\partial D_\theta}{\partial \theta} + \frac{\partial D_z}{\partial z} + \frac{D_r}{r} = 0, \end{cases} \quad (1)$$

where ρ is density of the piezoelectric tube; ξ_r, ξ_θ and ξ_z are the displacement components in the r, θ and z directions; D_r, D_θ and D_z are components of the electric displacement.

For the tube, the relationship between strain and displacement can be reduced to the following form:

$$\begin{bmatrix} S_r & S_\theta & S_z & S_{rz} \end{bmatrix} = \begin{bmatrix} \frac{\partial \xi_r}{\partial r} & \frac{\xi_r}{r} & \frac{\partial \xi_z}{\partial z} & \frac{\partial \xi_r}{\partial z} + \frac{\partial \xi_z}{\partial r} \end{bmatrix}. \quad (2)$$

In cylindrical coordinates, when the edge effect of the electric field is ignored, we have three components of the electric field $E_r = E_\theta = 0$, $E_z \neq 0$. The piezoelectric constitutive equations for the longitudinally polarized piezoelectric tube can be expressed as [20],

$$\begin{cases} T_r = c_{11}^E S_r + c_{12}^E S_\theta + c_{13}^E S_z - e_{31} E_z, \\ T_\vartheta = c_{12}^E S_r + c_{11}^E S_\theta + c_{13}^E S_z - e_{31} E_z, \\ T_z = c_{13}^E S_r + c_{13}^E S_\theta + c_{33}^E S_z - e_{33} E_z, \\ T_{rz} = c_{44}^E S_{rz}, \\ D_r = e_{15} S_{rz}, \\ D_\theta = 0, \\ D_z = e_{31} S_r + e_{31} S_\theta + e_{33} S_z + \varepsilon_{33}^S E_z \end{cases} . \quad (3)$$

Here, c_{ij}^E is the stiffness constant measured at constant electric field; e_{ij} is the piezoelectric stress constant and ε_{33}^S is the dielectric constant measured at constant strain.

At the piezoelectric tube ends $z = (0, l)$, with free boundary condition of both ends $T_z = 0$, the solution of Eq. (1) is the harmonic function of coordinate z and time t, which can be expressed as:

$$\begin{aligned} \xi_r &= u_r \sin(k_z z) e^{j\omega t} \\ \xi_z &= u_z \cos(k_z z) e^{j\omega t}. \end{aligned} \quad (4)$$

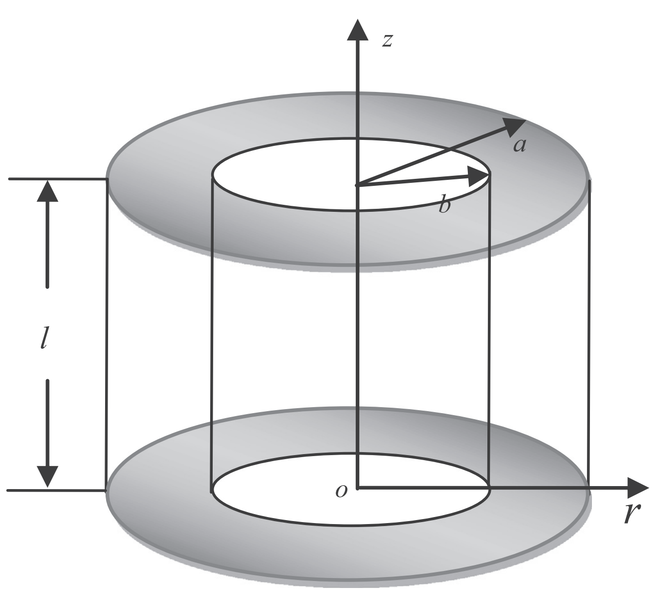

Fig. 1. Schematic diagram of a piezoelectric tube.

Here, $j = \sqrt{-1}$, ω is angular frequency, and k_z is axial wave number, $k_z = n\pi/l$ where n is an integer. Substituting Eqs. (2)–(4) into Eq. (1) and eliminating the electric field variable E_z, Eq. (1) can be expressed as follows:

$$\begin{cases} \frac{\partial^2 u_r}{\partial r^2} + \frac{1}{r} \cdot \frac{\partial u_r}{\partial r} - \frac{u_r}{r^2} + \left(\frac{\rho\omega^2}{c_{11}^E} - \frac{c_{44}^E}{c_{11}^E} \cdot k_z^2 \right) \cdot u_r - \frac{(c_{13}^E + c_{44}^E) \cdot k_z}{c_{11}^E} \cdot \frac{\partial u_z}{\partial r} = 0, \\ \frac{\partial^2 u_z}{\partial r^2} + \frac{1}{r} \cdot \frac{\partial u_z}{\partial r} + \frac{\varepsilon_{33}^S \rho\omega^2 - (c_{33}^E \varepsilon_{33}^S + e_{33}^2) k_z^2}{c_{44}^E \varepsilon_{33}^S + e_{33} e_{15}} \cdot u_z + \\ \frac{(\varepsilon_{33}^S (c_{13}^E + c_{44}^E) + e_{33}(e_{15} + e_{31})) k_z}{c_{44}^E \varepsilon_{33}^S + e_{33} e_{15}} \left(\frac{\partial u_r}{\partial r} + \frac{u_r}{r} \right) = 0 \end{cases} . \quad (5)$$

Solving for u_r and u_z yields [4]:

$$\begin{aligned} u_r &= -k_e[C_1 J_1(k_e r) + C_2 Y_1(k_e r)] + k_z[C_3 J_1(k_e r) + C_4 Y_1(k_e r)] \\ u_z &= k_z[C_1 J_0(k_e r) + C_2 Y_0(k_e r)] + k_s[C_3 J_0(k_e r) + C_4 Y_0(k_e r)]. \end{aligned} \quad (6)$$

Here, J_0 and J_1, Y_0 and Y_1 are Bessel functions of the first kind and the second kind. In Eq. (6), we have:

$$\begin{cases} k_e = \sqrt{\frac{k_1^2 + k_2^2 + B_r B_z k_z^2 + \sqrt{(k_1^2 + k_2^2 + B_r B_z k_z^2)^2 - 4k_1^2 k_2^2}}{2}}, \\ k_s = \sqrt{\frac{k_1^2 + k_2^2 + B_r B_z k_z^2 - \sqrt{(k_1^2 + k_2^2 + B_r B_z k_z^2)^2 - 4k_1^2 k_2^2}}{2}} \end{cases} .$$

where

$$k_1 = \sqrt{\frac{\rho\omega^2 - c_{44}^E k_z^2}{c_{11}^E}}, \quad k_2 = \sqrt{\frac{\varepsilon_{33}^S \rho\omega^2 - (c_{33}^E \varepsilon_{33}^S + e_{33}^2) k_z^2}{c_{44}^E \varepsilon_{33}^S + e_{33} e_{15}}},$$

$$B_r = \frac{(c_{13}^E + c_{44}^E)}{c_{11}^E}, \quad B_z = \frac{\varepsilon_{33}^S (c_{13}^E + c_{44}^E) + e_{33}(e_{15} + e_{31})}{c_{44}^E \varepsilon_{33}^S + e_{33} e_{15}}.$$

For the free vibration of the piezoelectric tube, the inner and outer boundary conditions can be written as:

$$\left. \begin{aligned} T_r|_{r=a} = 0, \quad T_r|_{r=b} = 0, \\ T_{rz}|_{r=a} = 0, \quad T_{rz}|_{r=b} = 0 \end{aligned} \right\}. \quad (7)$$

where

$$\begin{cases} T_r = \frac{e_{15} e_{31}}{\varepsilon_{33}^S} \cdot \frac{1}{k_z} \cdot \frac{\partial^2 u_z}{\partial r^2} + \frac{e_{15} e_{31}}{\varepsilon_{33}^S} \cdot \frac{1}{k_z r} \cdot \frac{\partial u_z}{\partial r} + \left(c_{11}^E + \frac{((e_{15} + e_{31}) e_{31})}{\varepsilon_{33}^S} \right) \frac{\partial u_r}{\partial r}, \\ + \left(c_{13}^E + \frac{((e_{15} + e_{31}) e_{31})}{\varepsilon_{33}^S} \right) \frac{u_r}{r} - \left(c_{13}^E + \frac{e_{33} e_{31}}{\varepsilon_{33}^S} \right) k_z \cdot u_z = 0, \\ T_{rz} = c_{44}^E k_z \cdot u_r + c_{44}^E \cdot \frac{\partial u_z}{\partial r} \end{cases} .$$

Substituting Eq. (6) into Eq. (7) yields the resonance frequency equation of longitudinal–radial coupled vibration for piezoelectric tube:

$$Eqs1 \cdot Eqs4 - Eqs2 \cdot Eqs3 = 0, \quad (8)$$

where

$$\begin{aligned} Eqs1 = {} & 2c_{44}^E k_e k_z \cdot J_1(k_e a) \cdot \frac{P \cdot eq1Y(a) + eq2J(a)}{eq1J(a)} \\ & - 2c_{44}^E k_e k_z \cdot Y_1(k_e a) \cdot P + c_{44}^E \left(k_z^2 - k_e k_s \right) \cdot J_1(k_e a), \end{aligned}$$

$$\begin{aligned} Eqs2 = {} & 2c_{44}^E k_e k_z \cdot J_1(k_e a) \cdot \frac{Q \cdot eq1Y(a) + eq2Y(a)}{eq1J(a)} \\ & - 2c_{44}^E k_e k_z \cdot Y_1(k_e a) \cdot Q + c_{44}^E \left(k_z^2 - k_e k_s \right) \cdot Y_1(k_e a), \end{aligned}$$

$$\begin{aligned} Eqs3 = {} & 2c_{44}^E k_e k_z \cdot J_1(k_e b) \cdot \frac{P \cdot eq1Y(a) + eq2J(a)}{eq1J(a)} \\ & - 2c_{44}^E k_e k_z \cdot Y_1(k_e b) \cdot P + c_{44}^E \left(k_z^2 - k_e k_s \right) \cdot J_1(k_e b), \end{aligned}$$

$$\begin{aligned} Eqs4 = {} & 2c_{44}^E k_e k_z \cdot J_1(k_e b) \cdot \frac{Q \cdot eq1Y(a) + eq2Y(a)}{eq1J(a)} \\ & - 2c_{44}^E k_e k_z \cdot Y_1(k_e b) \cdot Q + c_{44}^E \left(k_z^2 - k_e k_s \right) \cdot Y_1(k_e b), \end{aligned}$$