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Patch nearfield acoustic holography in a moving medium

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A B S T R A C T

To realize the accurate reconstruction of sound field in a moving medium under the condition of limited holographic aperture, a patch nearfield acoustic holography (NAH) in a moving medium is proposed. The proposed method not only reduces the influence caused by the limited aperture effects through sound field extrapolation, but also perfectly suits for sound field reconstruction in a moving medium by improving the shape of the modified Tikhonov regularization filter and the noise estimation method in accordance with flow effects. In the method, two cases that the flow direction is parallel to and perpendicular to the hologram surface are considered. Especially in the perpendicular case, the expression of the wavenumber component in the *z* direction is improved to make the proposed method suitable for the moving medium at a high Mach number. Simulations are investigated to examine the performance of the proposed method and show its advantages by comparing with NAH in a moving medium and the conventional patch NAH. It is found that, the proposed method is effective and robust at different flow velocities of the medium and different frequencies of the sound source.

1. Introduction

Nearfield acoustic holography (NAH) [1,2] is a powerful technique for identifying noise sources and visualizing the sound field by projecting the pressure (or particle velocity) measured by a microphone array (or particle velocity transducers) to the reconstruction surfaces. However, NAH was usually applied in the case that the sound sources, the microphone array and the medium are all static. When the sound sources are moving, the pressure measured by using the static microphone array will contain the frequency Doppler effect. To handle the frequency Doppler effect in the measured pressure, Kwon et al. [3] developed a moving frame technique and Yang et al. [4,5] proposed a method based on the point source assumption. In fact, the frequency Doppler effect is caused by the relative motion between the sound sources and the microphone array. So if the microphone array is moving synchronously with the sound sources, the measured pressure would not include the frequency Doppler effect any more, and instead, the flow effects resulted from the relative motion to medium will emerge. Actually, the moving-synchronously sound sources and microphone array are analogous to the stationary sound sources and microphone array in a moving medium. Ruhala et al. [6,7] derived the reconstruction formulas of NAH in a moving medium for two cases in which the flow directions were parallel to and perpendicular to the hologram surface, respectively, and the changes in the wavenumber domain caused by flow effects were also analyzed for both cases. In the former case, the highlighted change was that the radiation circle of defining the borderline between the propagating wave and the evanescent wave was distorted to a radiation ellipse. To simplify the shape of the wavenumber filter in NAH, Ruhala et al. [7] introduced a new radiation circle instead of the radiation ellipse by supposing that the particle velocity perpendicular to the flow direction was not influenced by the moving medium. In the latter case, based on the same assumption, Ruhala et al. [7] concluded that the radiation circle was unchanged compared to that in the static situation, and the only change was that the wave number increased in the upstream and decreased in the downstream. Kwon et al. [8] further analyzed the case that the flow direction was parallel to the hologram surface, and found that the simplification of the shape of the wavenumber filter restricted the NAH to low Mach numbers due to significant errors resulted from the high-speed moving medium. Therefore, they proposed a new wavenumber filter by establishing the mapping relationship between the radiation circle and the radiation ellipse, making NAH suitable for the moving medium at any subsonic velocity. In all the above-mentioned studies, the pressure measured by a microphone array was taken as the input of NAH. In order to avoid the disturbance of the airflow to microphones for the aerodynamic acoustic problem, Parisot-Dupuis et al.

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[9,10] applied the Laser Doppler Velocimetry to measure the particle velocity on the hologram surface non-intrusively, and then took the particle velocity as the input to reconstruct the sound field. However, this kind of measurement is relatively difficult, costly and time consuming.

Further considering the moving sound sources of large size, for example, the high-speed train and the cruising aircraft and so on, it is unpractical to cover these sound sources using a microphone array. Meanwhile, sometimes only a part of the sound source, such as the wheels of the train or the landing gears of the aircraft, is what we are interested in. In this case, a small microphone array covering the area of interest is enough. Unfortunately, the use of a small microphone array leads to the significant limited aperture effects, which reduce the reconstruction accuracy to a great extent. To suppress the influence caused by the limited aperture effects, several patch NAH methods [11–14] have been proposed. However, these methods are applied to the static medium, and their applications in a moving medium are rarely considered. Recently, Kim et al. [15] presented an improved statistically optimal NAH that was applicable in a moving medium, to reduce the influence caused by limited aperture effects in the case that the flow direction was parallel to the hologram surface. In the present paper, a patch NAH in a moving medium is to be proposed, where both cases that the flow direction is parallel to and perpendicular to the hologram surface are to be considered. By extrapolating the measured pressure on a small hologram aperture to a larger one, the hologram aperture is enlarged indirectly, and thus the reconstruction errors caused by the limited aperture effects can be reduced effectively. Different from the conventional patch NAH, in the process of extrapolation, the flow effects will be taken into account for improving the shape of the modified Tikhonov regularization filter and the noise estimation method. Besides, the assumption given by Ruhala et al. [7] in the case that the flow direction is perpendicular to the hologram surface will be reconsidered so that the proposed method can realize the accuracy reconstruction for the moving medium at a high Mach number.

2. NAH in a moving medium

From the convective wave equation, the pressure reconstruction formulation of NAH in a moving medium can be derived as [8]

$$P(k_x,k_y,\omega,z_s) = P(k_x,k_y,\omega,z_h)G^{-1}(k_x,k_y,\omega,z_h - z_s), \tag{1}$$

where k_x and k_y represent the wavenumber components in the x and y directions, respectively, ω represents the angular frequency, z_h and z_s represent the coordinates of the hologram surface and the reconstruction surface on the z axis, respectively, and the transfer function G is expressed by

$$G(k_x,k_y,\omega,z_h - z_s) = e^{ik_z(z_h - z_s)}. \tag{2}$$

In Eq. (2), k_z represents the wavenumber component in the z direction. Considering that the expression of k_z varies with the flow direction, therefore, two cases that the flow direction is parallel to and perpendicular to the hologram surface are discussed separately in the following.

In the case that the flow direction is parallel to the hologram surface, such as the medium flows along the x direction, the k_z can be expressed as [8]

$$k_z = \begin{cases} \sqrt{k^2 - (1-M^2)k_x^2 - 2kMk_x - k_y^2}, & (1-M^2)k_x^2 + 2kMk_x + k_y^2 \leqslant k^2 \\ i\sqrt{(1-M^2)k_x^2 + 2kMk_x + k_y^2 - k^2}, & (1-M^2)k_x^2 + 2kMk_x + k_y^2 > k^2 \end{cases}, \tag{3}$$

where k is the wave number defined as $k = \omega/c_0$, c_0 represents the sound speed in the static medium; M is the Mach number defined as $M = V/c_0$, in which V is the flow velocity and has $|V| < c_0$. Here, it is noted that when the medium flows along the positive x direction, V is positive, and negative conversely.

Eq. (3) indicates that the borderline of distinguishing the propagating wave and the evanescent wave is changed from a radiation circle in the static medium to a radiation ellipse in the moving medium. As shown in Fig. 1(a), when the medium flows along the negative x direction, the borderline can be characterized by a radiation ellipse, with the center $(-a, 0)$, the semimajor axis r_x and the semiminor axis r_y, where

$$a = \frac{kM}{1-M^2}, \quad r_x = \frac{k}{1-M^2}, \quad r_y = \frac{k}{\sqrt{1-M^2}}. \tag{4}$$

In the case that the flow direction is perpendicular to the hologram surface, i.e. the medium flows along the z direction, an analytical expression of k_z was derived in Ref. [7] based on an assumption that the particle velocity perpendicular to the flow direction was not influenced by the moving medium. The derived k_z indicated that the radiation circle was the same as the static one. However, this k_z is no longer valid when the fluid medium is moving at a high Mach number due to the given assumption. To obtain the k_z suitable for the high Mach number, its derivation is reconsidered in Appendix A without the assumption, and it is expressed as

$$k_z = \begin{cases} [-Mk \pm \sqrt{k^2 - (1-M^2)(k_x^2 + k_y^2)}]/(1-M^2), & (1-M^2)(k_x^2 + k_y^2) \leqslant k^2 \\ [-Mk \pm i\sqrt{(1-M^2)(k_x^2 + k_y^2) - k^2}]/(1-M^2), & (1-M^2)(k_x^2 + k_y^2) > k^2 \end{cases}, \tag{5}$$

where the "+" sign applies to the sound wave propagating in the positive z direction, and "−" sign applies to the sound wave propagating in the negative z direction. From Eq. (5), it can be found that the radiation circle is changed with an enlarged radius of $r = k/\sqrt{1-M^2}$, as shown in Fig. 1(b).

3. Patch NAH in a moving medium

For a moving sound source of large size, it is possible to perform the reconstruction directly in the partial region through a patch NAH procedure. As shown in Fig. 2, the medium moves at a velocity of V in the negative x direction (or in the negative z direction). The hologram surface H_0 and the reconstruction surface S_0 are conformal, and they are included in the extrapolation surfaces H+ and S+ correspondingly. The patch NAH procedure in a moving medium can be described as follows:

- Step 1: Properly zero-padding the measured pressure $p(H_0)$ to get the initial iteration pressure $p^{(0)}(H+)$, that is

$$p^{(0)}(H+) = \begin{cases} p(H_0) & (\vec{r} \in H_0) \\ 0 & (\vec{r} \in \Omega) \end{cases}, \tag{6}$$

where $\vec{r} = (x, y, z)$ represents the position vector of the measurement point, and Ω represents the region of H+ except H_0. In the current research, the size of the surface H+ is selected five times of the surface H_0.

- Step 2: Iteratively calculate the filtered pressure on the surface H+ by the following formula

$$\tilde{p}^{(i)}(H+) = F^{-1}[L(\alpha^{(i)})F(p^{(i)}(H+))], \tag{7}$$

where F and F^{-1} represent two-dimensional spatial Fourier transform and its inverse transform, respectively, $\tilde{p}^{(i)}(H+)$ represents the extrapolated pressure after the ith iteration, and $L(\alpha^{(i)})$ represents a low-pass filter with a filter factor α. Here, the low-pass