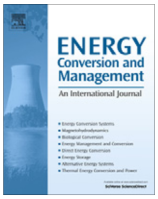
Energy Conversion and Management 65 (2013) 133–139

Contents lists available at SciVerse ScienceDirect

Energy Conversion and Management

journal homepage: www.elsevier.com/locate/enconman



Thermodynamic optimization of ground heat exchangers with single U-tube by entropy generation minimization method

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ARTICLE INFO

Article history:
Received 6 December 2011
Received in revised form 19 July 2012
Accepted 20 July 2012
Available online 16 October 2012

Keywords:
Ground-coupled heat pump system
Ground heat exchanger
Entropy generation minimization
Second-law-based analysis
Thermodynamic optimization

ABSTRACT

This paper investigates thermodynamic performance of borehole ground heat exchangers with a single U-tube by the entropy generation minimization method which requires information of heat transfer and fluid mechanics, in addition to thermodynamics analysis. This study first derives an expression for dimensionless entropy generation number, a function that consists of five dimensionless variables, including Reynolds number, dimensionless borehole length, scale factor of pressures, and two duty parameters of ground heat exchangers. The derivation combines a heat transfer model and a hydraulics model for borehole ground heat exchangers with the first law and the second law of thermodynamics. Next, the entropy generation number is minimized to produce two analytical expressions for the optimal length and the optimal flow velocity of ground heat exchangers. Then, this paper discusses and analyzes implications and applications of these optimization formulas with two case studies. An important finding from the case studies is that widely used empirical velocities of circulating fluid are too large to operate ground-coupled heat pump systems in a thermodynamic optimization way. This paper demonstrates that thermodynamic optimal parameters of ground heat exchangers can probably be determined by using the entropy generation minimization method.

1. Introduction

Currently there is much practical interest in designing ground-coupled heat pump systems (GCHPs), one of the best sustainable energy technologies [1–4], especially in developing countries such as China [1] and Turkey [2]. GCHPs use the ground as a heat source and sink as they transfer heat through buried ground heat exchangers (GHEs). Compared to ambient air, the ground provides low temperature for cooling and high temperature for heating, and the temperature fluctuation is low. Therefore, GCHPs offer an environment-friendly and energy-efficient way of providing cooling and heating, as well as hot water. The most used GHE is borehole GHEs, consisting of one or two U-tubes inserted into a vertical borehole and connected to a heat pump to form a closed loop (Fig. 1). Water with or without antifreeze is circulated in the closed loop. The space between the borehole wall and the U-tubes is filled with grout to enhance heat transfer between soil and the circulated fluid. Analysis of the process of heat transfer is the key to modeling and designing GHEs and GCHPs.

Analysis of heat transfer has proved very successful in design and simulation of borehole GHEs [5–11]. Several authors have reviewed recent advances in this field [1,12]; therefore, only works directly related to the present study are reviewed briefly. When analyzing heat transfer processes of borehole GHEs, a borehole is theoretically separated into two parts in space with its radius. Transfer of heat outside a borehole is a complicated three-dimensional unsteady process. Its solution gives the average temperature at the borehole wall, a temperature that connects these two parts of heat transfer analysis. Typically, radii of boreholes used for GHEs, r_b ($\sim$0.05 m), are very small compared to borehole length L ($\sim$100 m). So, heat conduction outside a borehole may be modeled as a line of heat sources or sinks of infinite or finite length that transfer heat to the surrounding soil [6,7]. This process can also be treated as unsteady heat conduction in an infinite region bounded internally by a circular cylinder [8]. All these models give average temperatures at the borehole walls, which are used for computing heat transfer inside boreholes.

In contrast, heat transfer inside a borehole is generally assumed to be in steady-state due to the much smaller dimensions and thermal capacity. Such assumption has been proved to be appropriate and convenient for most engineering design. Several models with varying complexity have been developed for this purpose. The simplest expression is the one-dimensional model treating a U-tube as a pipe with an "equivalent" diameter [9]. Hellstrom developed two two-dimensional models accounting for thermal influence

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http://dx.doi.org/10.1016/j.enconman.2012.07.013

Nomenclature

A	duty parameter
B	duty parameter
c	specific heat (J kg^{-1} K^{-1})
D	half spacing of U-tube (m)
f	friction factor
h	enthalpy (J kg^{-1})
L	length (m)
k	conductivity(W m^{-1} K^{-1})
$\dot{m}$	mass flow rate (kg s^{-1})
N_S	dimensionless entropy number
P	absolute pressure (Pa)
$\dot{Q}$	heat transfer rate (W)
r	radius (m)
R	thermal resistance per unit bore length (m K W^{-1})
s	entropy (J kg^{-1} K^{-1})
$\dot{S}_{\mathrm{gen}}$	entropy generation rate (W K^{-1})
T	thermodynamic temperature (K)
$\dot{W}$	work transfer rate (W)
Re	Reynolds number
Nu	Nusselt number

Pr	Prandtl number
ε	temperature efficiency
μ	dynamic viscosity (kg m^{-1} s^{-1})
ρ	density (kg m^{-3})
Θ	dimensionless temperature
Φ	scale factor of pressure
α	convective heat transfer coefficients (W m^{-2} K^{-1})

Subscript

0	environmental states
1	inlet of U-tube
2	outlet of U-tube
b	borehole/borehole wall
f	fluid
g	ground heat exchanger
i	inner
o	outer
m	mean
p	U-pipe
opt	optimal value

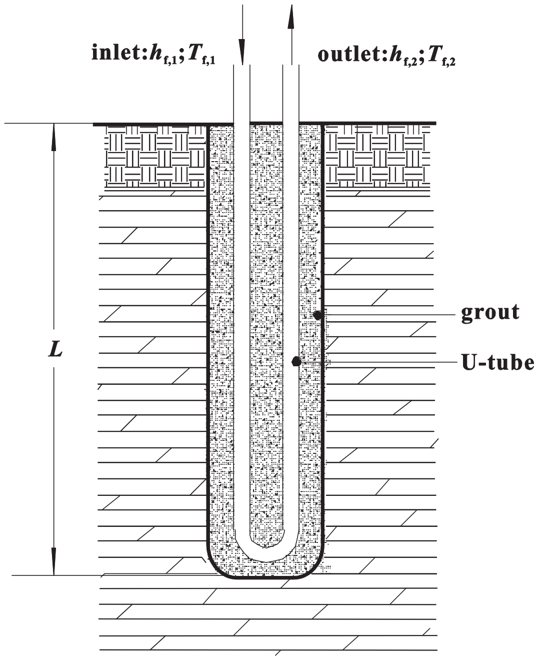

Fig. 1. Schematic description of a vertical ground heat exchanger with single U-tube.

between U-tube legs [10]; one is derived from the line-source assumption, and the other is derived from the multipole method. The difference between these two models is less than 10% and is insignificant for most engineering applications. Referring to Hellström's two-dimensional models, Zeng et al. [11] presented a quasi-three-dimensional model that can account for variation of fluid temperature along borehole depth.

Moreover, a composite-medium line-source model has been developed for borehole and pile GHEs [5], which can consider effect of grout materials in boreholes and property difference between ground and pile/grout materials. Despite this model being somewhat complicated, it may contribute to providing a new way of modeling short-term behaviors of vertical pile and borehole GHEs.

Recently, some works investigated thermodynamic performance of GCHPs by exergy analysis [12–15]. Exergy analysis consists of applying the first and the second laws of thermodynamics to evaluate the performance of system in the reversible limit and to estimate the departure from this limit [16]. Therefore, this analysis can be used to identify key potential energy saving components. Based on their exergy analysis, Bi et al. [12] showed that GHEs have the minimum exergy efficiency and thermodynamic perfection, which need to be improved. However, very few works have aimed at this problem.

To improve thermodynamic perfection of GHE, one has to consider not only thermal behavior of GHE but also hydraulic aspects. Hydraulic aspects of GHEs affect the friction irreversibility and operating cost of GCHPs, which should not be overlooked [17]. Generally, fluid flow in GHEs should be high enough to be turbulent, to augment the heat transfer. However, velocity of fluid cannot be kept too high since pressure drop should be as low as possible. Therefore, the way of determining the flow rate is an optimization problem. However, such criteria for determining optimum flow velocities in GHEs are still lacking, which have partly motivated this study. As is shown later, improper fluid velocities can result in waste of a large amount of energy and high operating cost of GCHPs. Reasonable optimization criterion can improve the overall performance of GCHPs greatly, counteracting the relatively high initial cost of GCHPs, further facilitating its application.

The purpose of this paper is to devise analytical expressions for optimizing flow velocity and bore length by using the entropy generation minimization (EGM) method. The EGM, or thermodynamic optimization, is a method of modeling and optimization of real devices with thermodynamic imperfections [16,18]. Minimizing entropy generation is equivalent to optimization of performance of thermal systems in the thermodynamic sense. Research on EGM has experienced tremendous growth during the last three decades, and it has been applied for optimization of power plants, fluid flow systems and electrical machines [16,18,19], etc. To the best of authors' knowledge, however, there is no published work on EGM analysis for GHEs. So this study is theoretically interesting also.