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Improvements for accuracy and stability in a weakly-compressible particle method

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ABSTRACT

In the Moving Particle Semi-implicit method (MPS), the original Laplacian model is introduced from a transient diffusion problem and a parameter is required to eliminate the error. Another Laplacian model is developed to mainly enhance the pressure calculation. When using an equation of state instead of the Poisson pressure equation to calculate pressure as weakly-compressible MPS (WC-MPS), the pressure noise problem is significant. In this study, a new Laplacian model is derived from the divergence of the original gradient model in MPS to improve the accuracy in the method. A stabilization technique developed from the continuity equation is proposed to enhance the incompressibility and to reduce pressure noise. The new Laplacian model is validated by a 2D diffusion problem and the Couette flow. It is able to calculate accurate results in the 2D diffusion problem with analytical solution. In the Couette flow, the accuracy of three models is compared and investigated with consideration of the interaction radius and the weighting function. It shows that the new Laplacian model is able to calculate more accurate results in the Couette flow. The weighting function plays an insignificant role in improving the accuracy of the new model in the problem. The stabilization technique is validated by a water jet impinging on a rigid flat plate. In the validation, effects from the collision model, repulsive force, weighting function, and particle distance are investigated. The stabilization technique is able to greatly reduce the pressure noise and unphysical pressure fluctuations. With the two techniques, the improved weakly-compressible MPS (IWC-MPS) is applied into modeling a dam-breaking flow. Comparisons show that the IWC-MPS attained good agreement with experimental measurements. Pressure noise and unphysical fluctuations are greatly eliminated. Compared with other numerical methods, IWC-MPS can obtain good results.

1. Introduction

The Moving Particle Semi-implicit method (MPS) and Smoothed Particle Hydrodynamics method (SPH) are useful numerical tools for studying free-surface flows in various engineering problems [1–9]. These mesh-free methods can easily simulate complicated fluid interface flows since the interface can be identified by tracing particles.

MPS was originally proposed by Koshizuka and Oka [10] for modeling viscous incompressible flows. Improvements have been made in applying MPS in simulating various types of flows [14–21]. As a Lagrangian method, continuum mechanics is analyzed in the concept of particles, which represent the fluid in MPS. This makes spatial differential operators or models calculated in simulations through summations over particles, which carry flow properties such as velocity and pressure. Two principal approaches can address the numerical implementation in obtaining pressure. If the Poisson equation is used to calculate the pressure field, the method is referred to as fully-incompressible MPS [1,2,10]. If the pressure field is obtained from solving an equation of state and incompressible fluids such as water are assumed to be weakly-compressible, the method is referred as Weakly-Compressible MPS (WC-MPS) [3,11,12]. Although the weakly-compressible scheme generates numerical oscillation in the pressure and density fields, it is attractive in parallel computing for simulations especially in cases involving large number of particles [13] and is considered an important particle method.

In MPS, there are two Laplacian models that have wide applications. The original Laplacian model, introduced by Koshizuka and Oka [22] was developed from a transient diffusion problem such that an extra parameter needs to be incorporated in order to ensure that the increase in variance is equivalent to the diffusion problem. However, inaccuracy still exists [23]. Khayyer and Gotoh [15] derived another model that was based on the divergence of the SPH gradient model. The derived Laplacian model focuses on the enhancement and stabilization of the pressure calculation than

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the accuracy of the viscous term discretization [15,18–20]. In fact, using the original gradient model to derive a new Laplacian model in MPS is still possible, which maintains consistency of general particle arrangement [24]. In this study, a new Laplacian model is derived with consideration of the original gradient model in MPS [10]. The new model is evaluated with respects to the particle distance, interaction radius, and weighting function. Its accuracy is compared with previous Laplacian models.

The particle number density as a normalized factor in MPS is proportional to the fluid density. However, it is constant when modeling the incompressible fluid flows. The discrepancy of the particle number density is generally observed in calculations [14,24]. Variation in the particle number density implies density discrepancy. Therefore, various types of source terms were developed for the Poisson equation in the fully incompressible MPS to enhance stability [14,16–18,20,25]. On the other hand, the collision model [17,21,25,26,30] and repulsive force model [16,17,22,27–29] are commonly applied in MPS to solve particle clustering or unphysical voids. These techniques may introduce particle penetration or random movements of particles if excessive repulsive force is produced [28,31], which makes the density discrepancy problem more complicated. In WC-MPS, the problem of density discrepancy is more significant since the incompressible fluid is treated as weakly compressible. To reduce the density discrepancy, a stabilization technique is developed to maintain the incompressibility of the incompressible fluid. By applying the newly developed technique, the density discrepancy could be rectified and particle penetration is minimized. Therefore, this technique is beneficial to WC-MPS in the stabilization of simulations.

The paper is organized as follows: in Section 2, WC-MPS formulation is presented; a new Laplacian model is developed and validated by a 2D diffusion problem and the Couette flow in Section 3; a stabilization technique is proposed and evaluated by a water jet impinging on a rigid plate with the consideration of the parameters such as the collision distance, repulsive force models, weighting function, and particle distance in Section 4; and Section 5 presents the application of the improved WC-MPS (IWC-MPS) including the new Laplacian model and the stabilization technique to simulate the dam-breaking flow and comparisons are made with experimental measurements to various numerical methods.

2. WC-MPS formulation

2.1. Governing equations

The governing equations for incompressible Newtonian fluids in the Lagrangian framework are:

$$\frac{1}{\rho}\frac{\mathrm{D}\rho}{\mathrm{D}t} + \nabla\cdot\boldsymbol{u} = 0$$
$$\frac{\mathrm{D}\boldsymbol{u}}{\mathrm{D}t} = -\frac{1}{\rho}\nabla p + \upsilon\nabla^2\boldsymbol{u} + \boldsymbol{F} \quad (1)$$

where ρ is the density, $\boldsymbol{u}$ is the velocity vector, t the time, p the pressure, υ the kinematic viscosity, and $\boldsymbol{F}$ is an external force (i.e., gravity). All vector quantities are written in bold and gradient operator is illustrated in "∇".

The predictor-corrector time-stepping scheme is adopted to solve the governing equations [10,30,32]. In the predictor, the viscosity and force terms are solved to obtain intermediate velocity $\boldsymbol{u}^*$ and intermediate particle position $\boldsymbol{r}^*$. In the corrector step, the pressure term is calculated to obtain the new velocity, and particle locations are updated by moving particles with the new velocity.

2.2. Weighting function

In MPS, the fluid and boundary domain is represented by a set of discretized particles. In the calculation, each fluid particle in the domain only interacts with neighboring particles in its interaction circle. This interaction circle is determined by a weighting function, also used to determine the contribution of each neighboring particle to the target particle. Neighboring particles located closer to the target particle have greater contribution than particles located farther away. There are two widely-used weighting functions in MPS applications WF1 [1,22,15,16] Eq. (2) and WF2 [3,11,12,30,32] Eq. (3), expressed as:

$$\text{WF1}: \quad W_{ij} = W\left(r_{ij}, r_e\right) = \begin{cases} \frac{r_e}{r_{ij}} - 1 & r_{ij} \le r_e \\ 0 & r_{ij} > r_e \end{cases} \quad (2)$$

$$\text{WF2}: \quad W_{ij} = W(r_{ij}, r_e) = \begin{cases} \left(1 - \frac{r_{ij}}{r_e}\right)^3 & r_{ij} \le r_e \\ 0 & r_{ij} > r_e \end{cases} \quad (3)$$

where W_{ij} is the value of a weighting function, i is the target particle, j is a neighboring particle, r_{ij} is the distance between the particle i and j, and r_e is the radius of the interaction circle.

WF1 has an infinite value at $r_{ij} = 0$. It means that a neighboring particle (the jth particle) has a greater contribution to the ith particle as the distance between them is decreased, while the value of WF2 is equal to 1.0 in the center. The first-order derivative of WF1 still has an infinite value in the center and it is equal to $-3.0/r_e$ at $r_{ij} = 0.0$ for WF2.

Particle number density as a normalized factor, which is proportional to the fluid density [10,14,22], is calculated as:

$$\langle n\rangle_i = \sum_{j\neq i} W(r_{ij}, r_e) = \sum_{j\neq i} W_{ij} \quad (4)$$

For an incompressible fluid, the density is constant, and the standard value n_0 calculated from the initial particle distribution is used to substitute $<n>_i$ in the following models [14,22].

2.3. MPS models

In MPS, the first gradient model, the Laplacian model (viscous model) and the divergence model were developed in association with the weighting function. The original gradient model was proposed by Koshizuka and Oka [10], written as:

$$\langle\nabla\phi\rangle_i = \frac{D_s}{n_0}\sum_{j\neq i}\frac{\phi_j - \phi_i}{r_{ij}^2}\boldsymbol{r_{ij}}W_{ij} \quad (5)$$

where D_s is the coefficient for dimension of space, $\boldsymbol{r_{ij}} = \boldsymbol{r_j} - \boldsymbol{r_i}$ is the position vector, $r_{ij} = |\boldsymbol{r_{ij}}|$ and ϕ is a general scalar.

In MPS, when the first gradient model is used for the pressure gradient [22], it is modified as:

$$\text{PGM1}: \quad \langle\nabla p\rangle_i = \frac{D_s}{n_0}\sum_{j\neq i}\frac{p_j - p_{i,\min}}{r_{ij}^2}\boldsymbol{r_{ij}}W_{ij}$$
$$= \frac{D_s}{n_0}\sum_{j\neq i}\left(\frac{p_j - p_i}{r_{ij}^2}\boldsymbol{r_{ij}}W_{ij} + \frac{p_i - p_{i,\min}}{r_{ij}^2}\boldsymbol{r_{ij}}W_{ij}\right) \quad (6)$$

where $p_{i,min} = \min_{j\subset J}(p_i, p_j) \quad J = \{j : W_{ij} \neq 0\}$.

This pressure gradient model is abbreviated as PGM1. PGM1 introduces an artificial repulsive force term (the second term on the right hand side of Eq. (6) for stabilization).

To achieve more repulsive force and the momentum conservation, two other pressure gradient models are respectively proposed by Khayyer and Gotoh [29] and Toyota et al. [27]: