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An improved lattice Boltzmann method for incompressible two-phase flows with large density differences

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ABSTRACT

We propose a new lattice Boltzmann method (LBM) for two-phase fluid flows with high density ratios by improving Inamuro et al.'s method [13] without solving the pressure Poisson equation. In the proposed method, we use the lattice kinetic scheme (LKS, an extended scheme of LBM) in the same way as Inamuro et al.'s method. The velocity and pressure fields are computed by using a single equilibrium distribution function and by adjusting the speed of sound in a high density region to satisfy the incompressible continuity equation even for high density ratios. In addition, an improved LKS is used for eliminating dissipation errors, and the continuous surface force (CSF) model is used for modeling interfacial tension of thin interfaces accurately. In order to show the validity of the method, we apply the method to the simulations of a stationary drop, binary droplet collision, rising bubbles, and the impact of a drop on a thin liquid film (a milk crown). In a stationary drop, pressure and density profiles are computed, and the effect of sound speed on time evolution of the pressure field in the drop is illustrated. The stable computations can be performed for high density ratios up to 855. In the simulations of a binary droplet collision and rising bubbles, the computed results by the proposed method are compared with those by Inamuro et al.'s method in good agreement. Also, the computation time of the proposed method is about 50 times faster than that of Inamuro et al.'s method . In the simulation of a milk crown, the time evolution of the crown radius is in good agreement with theoretical predictions.

1. Introduction

Two-phase fluid flows appear in many science and engineering fields. In particular, two-phase fluid flows with high density ratios are important in a lot of problems such as the behavior of drops and bubbles, surface waves on the ocean, spray injection, milk crown formation and so on. Many numerical methods have been proposed to simulate the two-phase fluid flows. For instance, the volume of fluid (VOF) method [8], the front tracking method [35], the level set method [22] and the diffuse interface methods (e.g. [2]) have been widely used. However, there are still some difficulties in keeping thin interfaces, mass conservation of each phase [34], and time-consuming computation of the Poisson equation for pressure. In addition, since the Poisson equation includes the coefficient of variable density under the divergence operator, careful treatments for solving the pressure equation are required especially for two-phase flows with high density ratios.

On the other hand, the lattice Boltzmann method (LBM) has been developed into an alternative and promising numerical scheme for two-phase fluid flows without solving the Poisson equation for the pressure (e.g. [1,11,31]). The LBM has an advantage over the above-mentioned methods in keeping thin interfaces and mass conservation of each phase. Also, because of its simplicity and computational efficiency as well as high scalability om parallel processing, the LBM has been applied to many simulations of two-phase fluid flows. However, usual LBMs for two-phase fluid flows (color-gradient models [7], pseudopotential models [27], and free-energy models [33]) cause numerical instability in computing the pressure for high density ratios (typically larger than 10). So far, many pseudopotential multiphase LBMs as well as color-gradient multiphase LBMs are proposed (e.g. [5,9]), but they usually have numerical instability for high density ratios, when fluid flows are relatively strong. Recently, Lycett-Brown et al. [19] simulated binary droplet collisions with the density ratio of approximately 120 using a multiphase cascaded LBM. Also, Mazloomi et al. [20,21] proposed a thermodynamically consistent LBM for two-phase flows using the entropic LBM and simulated binary droplet collisions with the density ratio of 110. In general, the simulation

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of a moving bubble is more unstable than that of liquid flows because of the numerical instability caused by the extremely small gas density [18]. A key issue of LBM for high density ratios is stable computing of the pressure field in incompressible flows including thin interfaces between liquid and gas phases.

In order to overcome the instability of LBM for high density ratios, several ideas for stable computing of the pressure field have been proposed mainly for free-energy models or phase-field models. Inamuro et al. [13] proposed a method for simulating two-phase fluid flows with high density ratios up to 1000 by solving the pressure Poisson equation. Lee and Lin [17] proposed a stable but complicated discretization scheme for the pressure and momentum evolution equations and simulated two-phase fluid flows with a density ratio of 1000. Both of the above approaches work well, but there is still some room for improvement in the pressure computing. For Inamuro et al.'s method, the pressure calculation without the Poisson equation is required, and simpler formulation and discretization for the pressure and momentum evolution equations are desirable for the method by Lee and Lin. Recently, Wang et al. [36,37] proposed a multiphase lattice Boltzmann flux solver (MLBFS) for incompressible multiphase flows with high density ratio. In their solver, the macroscopic flow variables at cell centers are computed by the finite volume method as in usual methods, and the fluxes at each cell interface are computed by LBM. The solver was validated by the simualtions of several benchmark two- and three-dimensional flows with density ratio up to 1000 and Reynolds number up to 3000. Shao and Shu [26] developed a hybrid phase field multiple relaxation time LBM and simulated two-dimensional two-phase flows with the density ratio of 1000, but the validations for three-dimensional two-phase flows with high density ratios are still required.

In the present paper, we propose a new simple LBM for two-phase fluid flows with high density ratios by improving the pressure computing of Inamuro et al.'s method without solving the pressure Poisson equation. In the proposed method, we use the lattice kinetic scheme (LKS, an extended LBM) [10] in the same way as Inamuro et al.'s method. The velocity and pressure field is computed by using a single equilibrium distribution function and by adjusting the speed of sound in a high density region. In addition, the improved LKS [32] is used for eliminating dissipation errors, and the continuous surface force (CSF) model [4] is used for modeling interfacial tension of thin interfaces accurately. In order to show the validity of the method, we apply the method to the simulations of a stationary drop, binary droplet collision, rising bubbles, and the impact of a drop on a thin liquid film (a milk crown).

The paper is organized as follows. In Section 2 we propose an improved lattice Bolzmann method for incompressible two-phase flows with large density differences. In Section 3 we present numerical examples in order to validate the proposed method. Finally, concluding remarks are given in Section 4.

2. Numerical method

Hereafter we use non-dimensional variables defined in Appendix A. In the same way as Inamuro et al. [13], we use the LKS (an extended scheme of LBM) [10] in which the relaxation time is set to unity and the macroscopic variables can be calculated without using the particle distribution function. In the LKS, a modeled gas, which is composed of identical particles whose velocities are restricted to a finite set of vectors, is considered as in the LBM. The three-dimensional lattice with fifteen velocity vectors (D3Q15 model) are used in the present study. The D3Q15 model has the velocity vectors $\boldsymbol{c}_i = (0, 0, 0)$, $(0, 0, \pm 1)$, $(0, \pm 1, 0)$, $(\pm 1, 0, 0)$, $(\pm 1, \pm 1, \pm 1)$ for $i = 1, 2, \ldots, 15$. The physical space is divided into a cubic lattice.

2.1. LKS for order parameter

The algorithm of computing the order parameter ϕ is the same as [13]. The time evolution of the order parameter $\phi(\boldsymbol{x}, t)$ at the lattice point $\boldsymbol{x}$ and time t is computed by the following equations:

$$\phi(\boldsymbol{x}, t + \Delta t) = \sum_{i=1}^{15} f_i^{\mathrm{eq}}(\boldsymbol{x} - \boldsymbol{c}_i \Delta x, t), \tag{1}$$

where Δx is a lattice spacing, Δt is the time step during which the particles travel one lattice spacing, and f_i^{eq} is an equilibrium distribution function. It is noted that $\Delta t = Sh\Delta x = O[(\Delta x)^2]$ where $Sh = \hat{U}_0/\hat{c} = O(\Delta x)$ is the Strouhal number as described in Appendix A. The equilibrium distribution function f_i^{eq} is given by

$$f_i^{\mathrm{eq}} = H_i\phi + F_i\left[p_0 - \kappa_f\phi\nabla^2\phi - \frac{\kappa_f}{6}|\nabla\phi|^2\right] + 3E_i\phi c_{i\alpha}u_\alpha + E_i\kappa_f G_{\alpha\beta}(\phi)c_{i\alpha}c_{i\beta}, \tag{2}$$

where $u_\alpha = O(\Delta x)$ is the fluid velocity computed in Section 2.2,

$$\begin{aligned} &E_1 = 2/9, E_2 = E_3 = E_4 = \cdots = E_7 = 1/9, \\ &E_8 = E_9 = E_{10} = \cdots = E_{15} = 1/72, \\ &H_1 = 1, H_2 = H_3 = H_4 = \cdots = H_{15} = 0, \\ &F_1 = -7/3, F_i = 3E_i (i = 2, 3, 4, \cdots, 15), \end{aligned} \tag{3}$$

and

$$G_{\alpha\beta}(\phi) = \frac{9}{2}\frac{\partial\phi}{\partial x_\alpha}\frac{\partial\phi}{\partial x_\beta} - \frac{3}{2}\frac{\partial\phi}{\partial x_\gamma}\frac{\partial\phi}{\partial x_\gamma}\delta_{\alpha\beta}, \tag{4}$$

with $\alpha, \beta, \gamma = x, y, z$ (subscripts α, β, and γ represent Cartesian coordinates and the summation convention is used). In the above equations, $\delta_{\alpha\beta}$ is the Kronecker delta and κ_f is a constant parameter of $O[(\Delta x)^2]$ determining the width of the interface.

In Eq. (2), p_0 is given by

$$p_0 = \phi T\frac{1}{1 - b\phi} - a\phi^2, \tag{5}$$

where a, b, and T are free parameters determining the theoretical maximum and the minimum of the order parameter $\phi_{\max}^{\mathrm{th}}$ and $\phi_{\min}^{\mathrm{th}}$. In a stationary flow field with a straight interface, the two phases correspond to $\phi_{\max}^{\mathrm{th}}$ and $\phi_{\min}^{\mathrm{th}}$, and the order parameter ϕ in the interface varies in the range of $\phi_{\min}^{\mathrm{th}} < \phi < \phi_{\max}^{\mathrm{th}}$. However, it is noted that in a dynamic flow field with a curved interface, the profile of ϕ around the interface changes a little with time from the stationary one.

The following finite-difference approximations are used to calculate the derivatives in Eqs. (2) and (4):

$$\frac{\partial\psi}{\partial x_\alpha} \approx \frac{1}{10\Delta x}\sum_{i=1}^{15} c_{i\alpha}\psi(\boldsymbol{x} + \boldsymbol{c}_i\Delta x), \tag{6}$$

$$\nabla^2\psi \approx \frac{1}{5(\Delta x)^2}\left[\sum_{i=2}^{15}\psi(\boldsymbol{x} + \boldsymbol{c}_i\Delta x) - 14\psi(\boldsymbol{x})\right]. \tag{7}$$

Note that the equations for the order parameter ϕ are the same as the free-energy model by Swift et al. [33].

The density in the interface is obtained by using the order parameter ϕ with the following relation:

$$\rho = \begin{cases} \rho_{\mathrm{G}}, & \phi = \phi_{\min}, \\ \dfrac{\Delta\rho}{2}\left[\sin\left(\dfrac{\phi - \overline{\phi}}{\Delta\phi}\pi\right) + 1\right] + \rho_{\mathrm{G}}, & \phi_{\min} < \phi < \phi_{\max}, \\ \rho_{\mathrm{L}}, & \phi = \phi_{\max}, \end{cases} \tag{8}$$

where ρ_{G} and ρ_{L} are the densities of gas and liquid phases, and $\phi_{\max}$ and $\phi_{\min}$ are the maximum and the minimum of ϕ in