Computers and Fluids 123 (2015) 1–9

Contents lists available at ScienceDirect

Computers and Fluids

journal homepage: www.elsevier.com/locate/compfluid

Penetrative convection of water in cavities cooled from below

Massimo Corcione*, Alessandro Quintino

DIAEE Sezione Fisica Tecnica, Sapienza Università di Roma, via Eudossiana 18, 00184 Rome, Italy

ARTICLE INFO

Article history:
Received 30 July 2014
Revised 27 July 2015
Accepted 14 September 2015
Available online 25 September 2015

Keywords:
Penetrative convection of water
Enclosure
Cooling from below
Numerical analysis
Dimensionless correlations

ABSTRACT

Transient natural convection in water-filled square enclosures with the bottom wall cooled at 0 °C, and the top wall heated at a temperature spanning from 8 to 80 °C, is studied numerically for different widths of the cavity in the hypothesis of temperature-dependent physical properties, starting from the initial condition of motionless fluid at the uniform temperature of the top wall. The sidewalls are assumed to be adiabatic. A computational code based on the SIMPLE-C algorithm is used to solve the system of the mass, momentum and energy transfer governing equations. The propagation of convective motion from the bottom toward the top of the enclosure is investigated up to the achievement of a steady-state or a periodically-oscillating asymptotic solution. It is found that the ratio between the penetration depth and the cavity size increases as the temperature of the heated top wall decreases and the cavity size increases. Moreover, when the configuration is such that the buoyancy force in the water layer confined between the cooled bottom wall and the density-inversion isotherm is of the order of that required for the onset of convection, the asymptotic solution is periodical. Finally, the coefficient of convection decreases with increasing both the cavity width and the imposed temperature difference. Dimensionless correlations are developed for the calculation of the heat transfer rate across the enclosure and the penetration depth.

1. Introduction

Penetrative convection is the propagation of convective motion from a lower convectively unstable fluid layer into an overlying contiguous stably stratified fluid region in which density increases from top to bottom. Other than occurring in several geophysical and astrophysical situations, penetrative convection takes also place in a horizontal water layer whose limiting boundaries are differentially heated above and below the maximum-density temperature of approximately 4 °C at atmospheric pressure, which our attention is focused on.

Besides the papers related to infinite horizontal layers with either stress-free or rigid top and bottom boundaries [1–6], the studies carried out on penetrative convection of water near its density-inversion point in cavities laterally confined by solid walls, which can be of interest for cooling thermal energy storage applications, are relatively few. The first of them was performed experimentally by Townsend [7] who used a tank having an ice-covered square bottom of 30 cm × 30 cm, filled with distilled water to a depth of 15 cm, whose upper surface was maintained at about 25 °C. Two runs were accomplished starting with water at 16 °C up to the achievement of the steady state. During each run, the evolutions of the mean temperature distribution and the downward heat flux were recorded. Moreover, observations of the motions of dye streaks and suspended particles were conducted, revealing the existence of a vigorous overturning in a thin layer adjacent to the cold bottom surface, from which rising columns of buoyant fluid emerged. These columns penetrated a constant-temperature region at nearly 3.2 °C occupying about two-thirds of the tank volume, up to being deflected horizontally as they reached the region of stable stratification. Temperature fluctuations were detected in the fluid layer just beyond the farthest penetration of the columns, apparently caused by internal waves excited by the impacts of the convective columns on the stable water layer. Later investigations were executed numerically on square enclosures by Zubkov and Kalabin [8], on cubic cavities by Zubkov et al. [9], and on shallow cylindrical containers by Li et al. [10], in all of which the heating-from-below condition was implemented assuming constant physical properties, except for the density in the buoyancy force term of the momentum equation. In particular, defined the density inversion parameter as $\theta_{inv} = (4\ °C - t_c) / (t_h - t_c)$, where t_h and t_c are the temperatures of the heated and cooled walls in Celsius degrees, Zubkov and colleagues assigned $\theta_{inv} = 0.5$ imposing $t_c = 0–3$ °C, whereas Li and co-workers fixed $\theta_{inv} = 0.3$, again imposing $t_c = 0–3$ °C. A number of partly related studies were recently performed by Li and colleagues on horizontal annular enclosures having more or less complex geometric configurations [11–16]. Primary aim of all these cited works was to determine the existence of multiple stable steady-state or periodically-oscillating flow pattern solutions. Other

* Corresponding author. Tel.: +390644585443; fax: +390649915905.
E-mail address: massimo.corcione@uniroma1.it (M. Corcione).

http://dx.doi.org/10.1016/j.compfluid.2015.09.003

Nomenclature

a_i	ith polynomial coefficient
c	specific heat at constant pressure, J/(kg K)
d_p	penetration depth, m
$\mathbf{g}$	gravity vector, m/s^2
H	distance of the 4 °C isotherm from the bottom wall in the perfectly stratified field, m
h	average coefficient of convection, W/(m^2 K)
$\mathbf{I}$	unit tensor
k	thermal conductivity, W/(m K)
Nu	Nusselt number
Pr	Prandtl number
p	pressure, Pa
Q	heat transfer rate, W
q	heat flux, W/m^2
Ra	Rayleigh number
T	period of oscillation, s
t	temperature, °C
U	horizontal velocity component, m/s
$\mathbf{V}$	velocity vector, m/s
V	vertical velocity component, m/s
W	width of the enclosure, m
x	horizontal Cartesian coordinate, m
y	vertical Cartesian coordinate, m

Greek symbols

δ_p	dimensionless penetration depth
φ	generic physical property
μ	dynamic viscosity, kg/(m s)
θ	dimensionless temperature
ρ	mass density, kg/m^3
τ	time, s
ψ	stream function, kg/(m s)

Subscripts

c	cooled bottom wall, at the temperature of the cooled bottom wall
h	heated top wall, at the temperature of the heated top wall
max	maximum value
0	at 0 °C

works with a bearing on the discussed topic are the numerical investigations carried out by Forbes and Cooper [17], Vasseur and Robillard [18] and Alawadhi [19] on transient natural convection of water in cavities whose boundary surface was totally or partially kept at 0 °C, for an initial water temperature $t_i \geq 4$ °C. Also in these studies the physical properties were assumed to be constant, accounting for the buoyancy effects by way of a proper temperature-density relation.

The above review of existing literature shows that the data available for the cooling-from-below configuration, and, consequently, for small values of the density inversion parameter, are very limited. Accordingly, no heat transfer correlating equation is available. For this reason, in the present paper a study of natural convection in water-filled square enclosures whose bottom wall is cooled at 0 °C, whereas the top wall is heated at a temperature ranging between 8 and 80 °C (which means that the density inversion parameter θ_{inv} defined above spans from 0.5 to 0.05), is performed numerically for cavity sizes in the range 1 – 10 cm, in the hypothesis of temperature-dependent physical properties, with the main scope to investigate the basic heat and momentum transfer features and develop a set of dimensionless correlations.

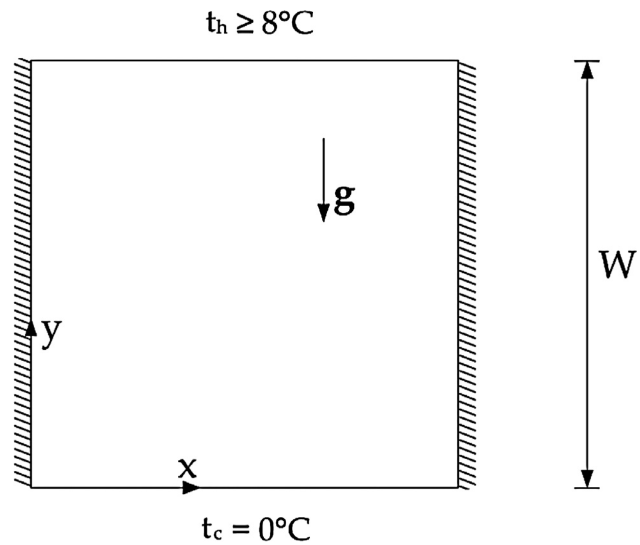

Fig. 1. Sketch of the geometry and coordinate system.

2. Mathematical formulation

A water-filled square enclosure of width W is considered. The enclosure is differentially heated at the bottom and top walls, which are kept at uniform temperatures $t_c = 0$ °C and $t_h \geq 8$ °C, respectively, while both sides are assumed to be perfectly insulated, as shown in Fig. 1, where the reference Cartesian coordinate system (x, y) is also represented. The consequent buoyancy-induced flow is considered to be two-dimensional, laminar and incompressible, with negligible viscous dissipation and pressure work.

In these hypotheses, the governing conservation equations of mass, momentum and energy reduce to

$$\frac{\partial \rho}{\partial \tau} + \nabla \cdot (\rho \mathbf{V}) = 0 \tag{1}$$

$$\frac{\partial (\rho \mathbf{V})}{\partial \tau} + \nabla \cdot (\rho \mathbf{V}\mathbf{V}) = -\nabla \cdot (p + \frac{2}{3}\mu \nabla \cdot \mathbf{V})\mathbf{I} + \nabla \cdot \mu[\nabla \mathbf{V} + (\nabla \mathbf{V})^t] + \rho \mathbf{g} \tag{2}$$

$$\frac{\partial (\rho c t)}{\partial \tau} + \nabla \cdot (\rho \mathbf{V} c t) = \nabla \cdot (k \nabla t), \tag{3}$$

where τ is the time, $\mathbf{V}$ is the velocity vector having horizontal and vertical components U and V, p is the pressure, t is the temperature in Celsius degrees, $\mathbf{g}$ is the gravity vector, ρ is the mass density, μ is the dynamic viscosity, c is the specific heat at constant pressure, k is the thermal conductivity, and $\mathbf{I}$ is the unit tensor. Superscript t indicates the transpose of $\nabla \mathbf{V}$.

The temperature-dependence of the generic physical property φ is approximated as a fourth-order polynomial function obtained by the best fit of 1001 equispaced reference data in the range 0 –100 °C extracted from the NIST Chemistry WebBook [20], put in the form:

$$\phi = \phi_0 \left(1 + \sum_{i=1}^{4} a_i t^i\right), \tag{4}$$

where φ_0 is the value of φ at the temperature of 0 °C, while a_1–a_4 are the polynomial coefficients given in Table 1. For any interpolation equation, the percentage standard deviation of the errors is of the order of 10^{-5}. As far as the mass density is specifically concerned, its temperature-distribution around the inversion point is represented in Fig. 2, in which selected values obtained by using the correlations