Computers and Fluids 123 (2015) 235–245

Contents lists available at ScienceDirect

Computers and Fluids

journal homepage: www.elsevier.com/locate/compfluid

An overlapping domain technique coupling spectral and finite elements for fluid–structure interaction

A.C. Verkaik[a,*], M.A. Hulsen[b], A.C.B. Bogaerds[c], F.N. van de Vosse[a]

[a] Department of Biomedical Engineering, Eindhoven University of Technology, the Netherlands
[b] Department of Mechanical Engineering, Eindhoven University of Technology, the Netherlands
[c] DSM Research, Materials Science Center, Geleen, the Netherlands

ARTICLE INFO

Article history:
Received 10 April 2015
Revised 4 October 2015
Accepted 8 October 2015
Available online 23 October 2015

Keywords:
Fluid–structure interaction
Spectral elements
Finite elements
Non-conforming meshes
Overlapping domains

ABSTRACT

In this study the coupled overlapping domain (COD) technique, previously developed within our group, is extended to make it suitable for fluid–structure interaction (FSI) computations with flexible elastic solids. A standard FSI benchmark from literature is implemented to verify the computations of the COD technique. However, in this study the incompressible neo-Hookean material model is applied instead of the compressible St. Venant-Kirchhoff model. The results of the COD technique are compared to the solutions obtained with a full finite element method. The performed computations demonstrate the COD technique accurately computes the solids displacements and the forces on the elastic solid. The results of the COD technique also show a good resemblance with the results obtained by other researchers, even with the use of a different material model. Additionally, the COD technique is applied to compute a rotating elastic beam in a square box filled with fluid. The computations show that the COD technique is capable of computing large deformations and translations, which makes this technique suitable for a wide range of applications without the need for computationally expensive remeshing.

1. Introduction

In a major part of the cardiovascular system blood flow is laminar, meaning blood flows mainly in parallel layers without lateral mixing. However, at some specific locations relatively high Reynolds number flows are observed, which causes the development of transitional flow, containing a mixture of laminar and turbulent flow. Based on the position and conditions in the cardiovascular system, the Reynolds number can become 4000 or even higher [1,2].

Computational methods can contribute to a better understanding of the transitional flow development at specific locations in the cardiovascular system, such as the flow in the left heart chamber [3–5] or flow around (artificial) heart valves [6–9]. Additionally, it can help to gain more insight into the development of hemodynamic complications or pathologies leading to or caused by transitional blood flow, for example flow around stenoses[10] and flow in abdominal aortic aneurysms [2,11].

Transitional blood flow in the cardiovascular system usually develops due to the interaction of high Reynolds number flows with a deforming solid. To obtain more insight into these complex interactions a fluid–structure interaction (FSI) method is needed, which is capable of computing accurately the interaction between the forces responsible for fluid flow and solid deformation. Therefore, the coupled overlapping domain (COD) technique has been developed which was introduced in Verkaik et al. [12]. The obtained results show the COD technique accurately computes the flow around rigid structures. The goal of this study is to extend the COD technique by implementing flexible and moving elastic structures. First, a short overview of the most relevant FSI methods with respect to this study is given.

A frequently used method for FSI computations is the Arbitrary Lagrangian–Eulerian (ALE) method. In fluid dynamics the Eulerian description is generally used, while in structural mechanics the Lagrangian description is preferred. The ALE method combines the advantages of both methods by allowing the computational mesh nodes to move in an arbitrary way, where the convective term of the fluid equations is modified to include the motion of these nodes [13–16]. If the fluid–structure interface of ALE methods is conformal, the strong fluid–solid coupling leads to accurate computations of the velocity, pressure and the interface location [17]. To compute fluid–structure interaction with a full finite element method (FEM), the ALE-method is applied to include the grid movement of the fluid domain. However, a disadvantage is that FSI methods with only ALE-correction are not suited for large translations and/or rotations, as this will lead to

* Corresponding author. Tel.: +31 40 247 3047; fax: +31 40 244 7355.
E-mail addresses: A.C.Verkaik@tue.nl (A.C. Verkaik), M.A.Hulsen@tue.nl (M.A. Hulsen), Arjen.Bogaerds@dsm.com (A.C.B. Bogaerds), F.N.v.d.Vosse@tue.nl (F.N. van de Vosse).

http://dx.doi.org/10.1016/j.compfluid.2015.10.007
0045-7930/

highly distorted meshes and consequently inaccurate computations. Eventually, computational expensive remeshing is required, which is associated with interpolation errors.

The COD technique is developed to circumvent this problem and to combine the advantages of spectral and finite elements. Spectral elements are able to obtain accurate flow computations for transitional flows [18,19], whereas finite elements are able to describe the possibly non-smooth elastic deformation of a flexible solid. The additional finite elements fluid layer, which is coupled conformal to the elastic solid, makes it possible to accurately compute the pressure jump and deformation rate of flexible solids moving in (transitional) fluid flow. The ALE-method is used to take the movement of the FEM fluid layer into account.

The COD technique is based on the Chimera method, where a structured background mesh covers the entire fluid domain and a separate domain is able to move freely on top of the background mesh [20,21]. It results in a great flexibility for computing moving bodies as well as for the generation of independent meshes, for example consisting of different type of elements, a different orientation or different mesh sizes for local mesh refinement. Nodes of the structured background mesh are activated or deactivated, depending on the position of the moving structure on top of the background mesh. Therefore, no active fictitious fluid exists underneath the overlapping domain, unlike the immersed boundary or fictitious domain methods, which has the risk of imposing artificial viscosity and incompressibility [22].

The Chimera methods applied to compute fluid–structure interaction, e.g. [21,22], perform a sequential iteration over the overlapping sub domains. In contrary, the COD technique couples spectral and finite elements in a monolithic FSI method, using a single system matrix to simultaneously solve the fluid and solid equations.

The goal of this study is to extend the earlier developed COD technique for FSI computations with flexible elastic solids. To verify and compare the obtained solutions of the COD technique with the solutions of a full FEM method an FSI benchmark was computed, based on the FSI benchmark developed by Turek and Hron [23]. Furthermore, the COD technique was applied to compute a rotating elastic beam in a square box filled with fluid, to investigate if the COD technique is capable of computing large displacements and deformations without remeshing.

This paper is structured as follows. Section 2 describes the governing equations for the fluid and elastic solid. In Section 3 the COD technique is explained and a short description of the full FEM method is given. Section 4 describes the FSI benchmark and the results obtained for this benchmark with the COD technique and the full FEM method. In Section 5 the application of a rotating beam in a squared box filled with fluid is presented and the obtained results are shown. Section 6 contains the discussion about the results of the adapted FSI benchmark and the rotating elastic beam. Section 7 contains the general conclusions of this study.

2. Governing equations

For this study, fluid–structure interaction problems are considered between a Newtonian fluid and an elastic (neo-Hookean) solid. In Fig. 1 a schematic representation is given.

The fluid domain Ω_f is described by the incompressible Navier–Stokes equations in an Eulerian formulation by applying the ALE method taking into account moving grids:

$$\rho_f \left.\frac{\partial \boldsymbol{v}}{\partial t}\right|_{\hat{\boldsymbol{x}}} + \rho_f(\boldsymbol{v} - \boldsymbol{v}_{grid}) \cdot \nabla \boldsymbol{v} = \nabla \cdot \boldsymbol{\sigma} + \boldsymbol{f}, \tag{1}$$

$$\nabla \cdot \boldsymbol{v} = 0, \tag{2}$$

where $\left.\frac{\partial(\cdot)}{\partial t}\right|_{\hat{\boldsymbol{x}}}$ denotes that the time derivative is taken with respect to fixed reference coordinates $\hat{\boldsymbol{x}}$ of the fluid domain, $\boldsymbol{v}$ is the fluid

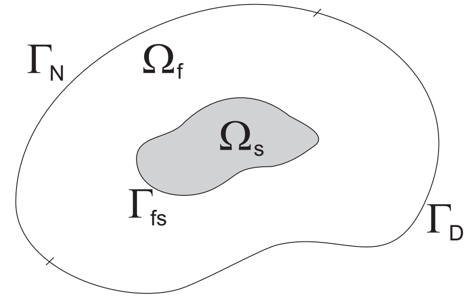

Fig. 1. A visualization of the domains, with Ω_f the fluid domain and Ω_s the elastic solid domain. Γ_{fs} is the boundary between the fluid and solid domain. The Dirichlet and Neumann boundary conditions are imposed on respectively Γ_D and Γ_N.

velocity, $\boldsymbol{v}_{grid}$ is the grid velocity, ρ_f is the fluid density, $\boldsymbol{f}$ a volumetric body force and $\boldsymbol{\sigma}$ the Cauchy stress tensor. The stress tensor is given by the constitutive equation for a Newtonian fluid

$$\boldsymbol{\sigma} = -p\boldsymbol{I} + 2\eta \boldsymbol{D}(\boldsymbol{v}), \tag{3}$$

with p the hydrostatic pressure, $\boldsymbol{I}$ the unity tensor, η the dynamic viscosity and $\boldsymbol{D}$ the rate of deformation tensor

$$\boldsymbol{D}(\boldsymbol{v}) = \frac{1}{2}\left(\nabla \boldsymbol{v} + (\nabla \boldsymbol{v})^T\right). \tag{4}$$

The incompressible elastic solid Ω_s is described by

$$\rho_s \frac{D^2 \boldsymbol{u}}{Dt^2} = \nabla \cdot \boldsymbol{\sigma}, \tag{5}$$

$$\det(\boldsymbol{F}) = 1, \tag{6}$$

in a Lagrangian formulation, where ρ_s is the solid density, $\boldsymbol{u}$ the solid displacement, $\boldsymbol{F} = (\boldsymbol{\nabla_0 x})^T$ is the deformation tensor with respect to initial configuration at $t = 0$, with $\boldsymbol{x}$ the position vector of the solid. The stress tensor for the neo-Hookean solid $\boldsymbol{\sigma}$ is described by

$$\boldsymbol{\sigma} = -p\boldsymbol{I} + G(\boldsymbol{F} \cdot \boldsymbol{F}^T - \boldsymbol{I}), \tag{7}$$

in which p represents the hydrostatic pressure within the solid and G is the shear modulus.

The following boundary conditions are prescribed for the outer most boundaries of the complete domain

$$\boldsymbol{v} = \boldsymbol{v}_D(t) \qquad \text{on} \quad \Gamma_D \tag{8}$$

$$-p\boldsymbol{n} + 2\eta \boldsymbol{D}(\boldsymbol{v}) \cdot \boldsymbol{n} = \boldsymbol{t}_N(t) \qquad \text{on} \quad \Gamma_N, \tag{9}$$

with Γ_D the Dirichlet boundary and Γ_N the Neumann boundary where the traction $\boldsymbol{t}_N$ acts on and $\boldsymbol{n}$ the outer normal to Γ_N.

On the fluid–structure interface (Γ_{fs}) the following kinematic and dynamic boundary conditions are imposed

$$\boldsymbol{v}_{|f} = \boldsymbol{v}_{|s} \qquad \text{on} \quad \Gamma_{fs} \tag{10}$$

$$\boldsymbol{\sigma}_{|f} \cdot \boldsymbol{n} = \boldsymbol{\sigma}_{|s} \cdot \boldsymbol{n} \qquad \text{on} \quad \Gamma_{fs}, \tag{11}$$

with $\boldsymbol{v} = D\boldsymbol{u}/Dt$ and $\boldsymbol{n}$ defined as the normal vector on Γ_{fs} pointing from the fluid domain into the solid domain. Resulting in a 'no slip'-condition for the flow and a force equilibrium on Γ_{fs}. All the computations for the fluid and solid start from rest, i.e. $\boldsymbol{v}_{|t=0} = \boldsymbol{0}$, $\boldsymbol{u}_{|t=0} = \boldsymbol{0}$ and $\left(\frac{D\boldsymbol{u}}{Dt}\right)_{|t=0} = \boldsymbol{0}$.

3. Methods

First, a complete overview of the COD technique is given, which was introduced in Verkaik et al. [12] and in this study is extended to compute FSI for flexible solids. Furthermore, some basic information is given about the applied full FEM method.