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# Study of irregular behavior of shear waves in layered soil using matrix and finite element methods

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ABSTRACT

This study uses both the theoretical matrix and finite element methods to simulate the three-dimensional (3D) wave propagation in elastic layered soils with a harmonic point load acting on the surface. Choosing different multi-layer cases (two, four and eight layers) where the point load is in horizontal or vertical direction, we first investigated the accuracy of the two methods, and the comparisons indicate that the results from both are in good agreement. Few authors have investigated the irregular wave amplitude of the Love wave induced in layered soils. This study indicates that the Love wave, unlike the Rayleigh wave, might generate larger ground vibrations for a wave located far away from the source, which is called wave hump in this paper. A ratio of the Young's modulus between the top and bottom soil layers larger than three may cause obvious this condition. Moreover, a layer thickness between 0.5 and 1.5 times the wave length in the first soil layer can significantly change the magnitude of the wave hump.


## 1. Introduction

Wave propagation in layered soil is an important but complicated problem. There are a number of theoretical matrix and numerical solutions [1–7] in assessing the issue. However, they often focus on the surface wave propagation induced by vertical loading, and not many studies relate to horizontal loading [8,9]. Moreover, the irregular magnitudes of surface waves in layered soils, including Rayleigh and Love waves, are rarely discussed in the literature. Although the medium for propagation is focused on the layered soil in this paper, it is worth mentioning that the complicated characteristic of wave propagation is existed not only in layered soil but also in other medium, such as the ocean (fluids) with a poroelastic seabed. Based on the Biot's theory, investigation of the elastic wave propagation in a fluid-saturated porous solid has also been performed in the topic of ocean acoustics. Several researches are presented in this paper. Gilbert and Lin [10] investigate the propagating solutions of the acoustic equation in a stratified shallow ocean with a poroelastic, semi-infinite seabed. Schmidt and Jensen [11] presented a numerical solution technique for wave propagation in horizontally stratified viscoelastic media using the Thomson–Haskell solution technique. Buchanan and Gilbert [12] developed a system of differential equations derived from Biot's constitutive and motion equations for a poroelastic material in a one-layer seabed. Obrezanova and Rabinovich [13] investigated sound propagation from moving sources in stratified waveguides by deriving an asymptotic representation for the acoustic field generated by a moving point source. Kumar and Hundal [14] derived a frequency equation connecting the phase velocity with wave number to estimate the surface wave propagation in a fluid saturated incompressible porous half-space lying under a double-layer. All the above references regarding ocean waves found that the pressure is not uniformly decayed, which means that a larger pressure can be measured while the distance from the source is more distant. However, the above phenomenon has not been clearly investigated for the wave propagation in soils, although some references [15–17] contained numerical or experimental results representing this event.

Layered soils under external force might generate surface vibration caused by wave propagation, and the complicated dynamic behavior of layered soils arises due to the interactions of reflection and refraction of waves at the medium interface of the layers. Therefore, the greater the number of layers, the more complicated the behavior. In the literature, the solution usually contains two or three soil layers, and not more than four. In this study, we utilized the finite element method and theoretical matrix method modified from Jones and Petyt [4] to simulate the wave propagation in layer soils with harmonic vertical and horizontal point loads acting on the surface, and the accuracy of the two methods for problems with two to eight soil layers were first examined. Then, the irregular surface wave propagations, including Rayleigh and Love waves, in layered soils was investigated in this study.

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## 2. Modeling and derivation of simulation methods

The problem is a half infinite domain with layered soils, and a 2a (=0.6 m) by 2b (=0.6 m) square area on the ground is subjected to a sine wave load with the magnitude ($p$) of 1 N in the $Y$ or $Z$ direction, where the negative $Z$ direction is the direction of gravity. The two methods, theoretical matrix and finite element, are described below.

### 2.1. Theoretical matrix method

To investigate the steady state vibration behavior of the wave propagation, Navier's elastodynamic equation neglecting the body force is used, as follows:

$$(\lambda+\mu)\frac{\partial\Delta}{\partial x_i}+\mu\nabla^2 u_i+\rho\omega^2 u_i=0\,,\quad i=1\sim 3 \tag{1}$$

$$\Delta=\nabla\cdot\mathbf{u}=\sum\frac{\partial u_i}{\partial x_i} \tag{2}$$

where $x_i$ and $u_i$ are the $i$th components of the Cartesian coordinate vector ($X$, $Y$, $Z$) and displacement vector ($u$, $v$, $w$), and the complex lamb constants are defined as follows:

$$\lambda=\frac{vE(1+\mathrm{i}\eta)}{(1+v)(1-2v)},\quad \mu=\frac{E(1+\mathrm{i}\eta)}{2(1+v)},\quad i=\sqrt{-1} \tag{3}$$

where $\eta$ is the loss factor, $v$ is the Poisson's ratio, and $E$ is Young's modulus. Using the derivation in Kramer [18], one can obtain that $\eta = 2D$, in which $D$ is the damping ratio. Differentiating Eq. (1) by each component of the vectors ($x$, $y$, $z$) individually, one obtains:

$$(\lambda+\mu)\frac{\partial^2\Delta}{\partial x_i^2}+\mu\left(\frac{\partial}{\partial x_i}\right)\nabla^2 u_i+\rho\omega^2\left(\frac{\partial}{\partial x_i}\right)u_i=0\,,\quad i=1\sim 3 \tag{4}$$

Treating each component of Eq. (4) using the double Fourier transform represented by notation $\mathfrak{F}$, summing equations of each component $i$, and rearranging the items, we obtain:

$$(d^2/dz^2-\beta^2-\gamma^2+\phi_1^2)\hat{\Delta}=0 \tag{5}$$

where $\hat{\Delta}=\mathfrak{F}\Delta$, $\phi_1^2=\frac{\omega^2}{c_p^2}$, $c_p^2=\frac{\lambda+2\mu}{\rho}$, and $\beta$ and $\gamma$ are wave numbers in the $x$ and $y$ directions, respectively. After the ordinary differential Eq. (5) is solved, the following general solution can be obtained:

$$\hat{\Delta}=Ae^{-\alpha_1 z}+Be^{\alpha_1 z},\quad \alpha_1=(\beta^2+\gamma^2-\phi_1^2)^{1/2} \tag{6}$$

where $\alpha_1=(\beta^2+\gamma^2-\phi_1^2)^{1/2}$, $A$ and $B$ are the constants of integration. Eq. (4) is then applied to the double Fourier transform, and $\hat{\Delta}$ is replaced by the right side of Eq. (6). Eq. (7) can thus be determined as follows:

$$\left(\frac{d^2}{dz^2}-\alpha_2^2\right)\begin{Bmatrix}\hat{u}\\ \hat{v}\\ \hat{w}\end{Bmatrix}=\left(-\frac{\lambda+\mu}{\mu}\right)\begin{pmatrix}\mathrm{i}\beta(Ae^{-\alpha_1 z}+Be^{\alpha_1 z})\\ \mathrm{i}\gamma(Ae^{-\alpha_1 z}+Be^{\alpha_1 z})\\ \alpha_1(-Ae^{-\alpha_1 z}+Be^{\alpha_1 z})\end{pmatrix} \tag{7}$$

where $\alpha_2^2=\beta^2+\gamma^2-\phi_2^2$, $\phi_2^2=\frac{\omega^2}{c_s^2}$, and $c_s^2=\frac{\mu}{\rho}$

The general solution of Eq. (7), named transformed displacement, can be written as

$$\hat{U}=\{\hat{u},\hat{v},\hat{w}\}^T=\hat{U}_h+\hat{U}_p \tag{8}$$

The subscripts $h$ and $p$ are homogenous and particular solutions, respectively. To find the solution of the ordinary differential Eq. (7), we introduce the stress–strain relations and take their double Fourier transform into the wavenumber domain. Two kinds of boundary conditions on the surface are defined as: (1) surface out loading in the $Z$ (vertical) direction, and (2) surface out loading in the $Y$ (horizontal) direction.

$$\hat{\tau}_{iz}|_{z=0}=-\frac{p}{2\pi ba}\frac{\sin(\beta b)}{\beta}\frac{\sin(\gamma a)}{\gamma} \tag{9}$$

The subscript $i$ in Eq. (9) corresponds to the loading direction. After this operation, we can obtain the algebraic relationship among the displacements $\hat{U}$, the stresses $\hat{\tau}$, and the square matrix $\{T\}$ whose coefficients are composed of both the soil parameters and the wavenumbers for each layer. i.e.

$$\{T\}_{6*6}[\hat{U}]_{6*1}=[\hat{\tau}]_{6*1} \tag{10}$$

Regarding the matrix $\{T\}$ of each layer as a 6 by 6 sub-matrix, and assembling and rearranging both the row and column numbers of each layer, $\{T\}|_{i^{th}\_layer}$ can be combined into the global soil matrix $\{K\}|_g$ of the whole layers based on the assumptions of continuity in the interface of each layer and boundary conditions, and the global transformed displacements for all layers can be written as Eq. (11):

$$[\hat{U}]_g=\{K\}_g^{-1}[\tau]_g \tag{11}$$

Finally, the inverse double Fourier transform of the vector $[\hat{U}]_g$ is calculated to obtain the displacement vector $[U]_g$ of each layer, i.e., the 3D displacements of all layers.

### 2.2. Finite element method

Due to the symmetry of the problem, a half finite element model is generated with the symmetry along the surface of $X = 0$, where rollers are set on that surface. The Newmark direct integration method with the average acceleration was used to solve this problem with the solution scheme of the SSOR (Symmetric Successive Over-Relaxation) preconditioned conjugated gradient method [19]. The finite element model is 30.75 m, 48 m, and 20 m in the $X$, $Y$, and $Z$ directions generated with the isoparametric 8-node solid elements. The solid element size is 0.15 m and the consistent mass scheme is used. The time step length is 0.01 s, with 4096 time steps simulated. Rayleigh damping was used in this study, and the two factors of $\alpha$ and $\beta$ ([Damping] = α[Mass] + β[Stiffness]) for the soil equal 8.0085/s and $1.9915\times10^{-4}$ s, respectively, which gives a 5% damping ratio at a frequency of 16 Hz and 64 Hz. If another damping ratio is used, the two factors are modified proportionally. Excluding the top surface and the symmetric surface (leftmost surface), the absorbing boundary conditions are set along the other four surfaces to avoid fake reflected waves on the mesh boundaries. The theory of the absorbing boundary condition is explained as follows [20]:

$$\left(\frac{\partial}{\partial t}-c_i\frac{\partial}{\partial x_1}\right)u_i=0\quad\text{for } i=1,\ 2\text{ and }3. \tag{12}$$

where $x_1$ is the coordinate with the positive direction pointing into the domain, while $x_1 = 0$ is at the absorbing boundary, $u_i$ is the $x_i$-displacement at the boundary, and $c_i$ is the $i$-direction velocity over the cosine of the incidence angle, and it can be evaluated as follows [20]:

$$c_i=\sqrt{\left(\sum_{k=1}^{N}\frac{\partial u_{ik}}{\partial t}\frac{\partial u_{ik}}{\partial t}\right)\Bigg/\left(\sum_{k=1}^{N}\frac{\partial u_{ik}}{\partial x_1}\frac{\partial u_{ik}}{\partial x_1}\right)}\quad\text{for}\quad i=1,\ 2\text{ and }3 \tag{13}$$

where $N$ is the selected nodes near the boundary, and $u_{ik}$ is $x_i$-displacement at the $k$th selected node. The forward Euler method deriving from Eq. (12) is then

$$u_{i,n+1}=u_{i,n}+\Delta tc_i\frac{\partial u_{i,n}}{\partial x_1} \tag{14}$$

where $u_{i,n}$ is the $i$-direction displacement at the last time step, $u_{i,n+1}$ is the $i$-direction displacement ($u_i$) at the current time step, and $\Delta t$