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Influence of magnetic field on the onset of nanofluid convection induced by purely internal heating

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ABSTRACT

This note considers the effect of magnetic field on the onset of convection in a nanofluid layer induced by purely uniform internal heating. The nanofluid layer bounded between two rigid surfaces and also incorporates the effect of Brownian motion along with thermophoresis. The zero nanoparticle flux condition under the thermophoretic effects is considered at the boundaries. The stability condition are found for two sets of thermal boundary conditions namely, case (i) both boundaries isothermal and case (ii) lower insulated and upper isothermal using the higher order Galerkin method. The purely internal heating problem shows that there is no applied temperature difference across the layer and so the external Rayleigh number is no longer appropriate. Therefore, here the relevant parameter is an internal Rayleigh number, one based on the heat source strength. It is found that the critical internal Rayleigh number increases with an increase in the magnetic Chandrasekhar number, while decreases with an increase in the Lewis number, the nanoparticle Rayleigh number and the modified diffusivity ratio. A comparative study between the previously published results and the present results for a special case is found to be in good agreement.

1. Introduction

In the recent decade, nanofluids exert a pull on a great deal of interest with their large potential to provide enhanced performance properties, mainly with respect to heat transfer. Nanofluids are used for cooling of microchips in computers and other electronics which use microfluidic applications [1–3]. Using nanofluids as coolants would allow for the radiators with smaller sizes and better positioning. The term nanofluid refers to these kinds of fluids by suspending nanometer sized metallic particles in common fluids [4]. The presence of nanoparticles in the fluids noticeably increases their effective thermal conductivity and consequently enhances their heat transfer characteristics. An overview of the literature dealing with recent developments in the study of heat transfer using nanofluids can be found in the articles [5–7].

The study of convective instability of the nanofluids is not only of academic interest but has many practical applications in chemical engineering, geophysics and astrophysics etc [8]. Buongiorno [9] proposed a model for convective transport in nanofluids incorporating the effects of Brownian diffusion and thermophoresis. This model was applied to study the thermal instability problem by Tzou [10,11] and observed that nanofluid is less stable than regular fluid. Later, this problem was revisited by Nield and Kuznetsov [12] by taking different types of non-dimensional parameters. Convective problem for rotating nanofluid was studied by Yadav et al. [13–15] and Chatterjee et al. [16]. Thermal conductivity and viscosity variation on the onset of convection in nanofluid was studied by Nield and Kuznetsov [17], Yadav et al. [18–20] and Umavathi et al. [21]. They obtained that the consequence of these factors increase the critical Rayleigh number. The effect of internal heat source on the onset of nanofluid convection was examined by Yadav et al. [22] and Nield and Kuznetsov [23]. They found that the basic temperature distribution and the basic volumetric fraction of nanoparticle distribution deviate from linear to non-linear in the presence of internal heating, and the critical Rayleigh number decreases with an increase in the internal heat source strength. Effect of magnetic field on nanofluid convection was considered by Sheikholeslami et al. [24–26], Al-Zamily [27], Yadav et al. [28,29], Gupta et al. [30], Hamada et al. [31] and Bansal and Chatterjee [32]. The common finding of these studies was that the fluid experienced a Lorentz force. This force, in turn, affects the buoyant flow field and the heat transfer rate.

The problem of convection induced by internal heating of an electrically conducting fluid, in the presence of an applied magnetic field, is of general interest from the viewpoint of many applications. Some of the wide application areas we have come across are in the metal casting, the cooling systems of electronic devices, MHD power generators, crystal growth in fluids and energy storage [33–36], but the

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Nomenclature

a	dimensionless wave number
a_c	critical wave number
c	specific heat
D_B	Brownian diffusion coefficient
D_T	thermophoretic diffusion coefficient
$\hat{\mathbf{e}}_\mathbf{z}$	unit vector in z-direction
$\tilde{\mathbf{g}}$	acceleration due to gravity
$\tilde{\mathbf{H}}^*$	magnetic field
$\tilde{\mathbf{j}}_p$	diffusion mass flux for the nanoparticle
k	thermal conductivity
L	thickness of nanofluid layer
L_e	Lewis number
N_I	modified diffusivity ratio
$\mathrm{N_B}$	modified specific heat increment
p	pressure
P_r	Prandtl number
P_{rM}	magnetic Prandtl number
Q	magnetic Chandrasekhar number
R_I	internal Rayleigh number
$R_{I,c}$	critical internal Rayleigh number
S_0^*	strength of internal heat source
t	time
T	temperature
$\tilde{\mathbf{v}}$	velocity of nanofluid
(x, y, z)	space co-ordinates

Greek symbols

α	thermal diffusivity
β	coefficient of thermal expansion
μ	viscosity
σ	electrical conductivity of the nanofluid
μ_e	magnetic permeability of the nanofluid
ρ	density of the nanofluid
ρ_0	reference density of nanofluid
ρ_p	density of nanoparticles
(ρc)	heat capacity
ϕ	volume fraction of the nanoparticles
ϕ_0^*	reference scale for the nanoparticle fraction
ζ_z	z-component of current density
∇_P^2	horizontal Laplacian operator
∇^2	Laplacian operator

Superscripts

*	non-dimensional variables
'	perturbed quantities

Subscripts

p	particle
b	basic state

research work in this field is limited. Convection induced by internal heating in electrically conducting fluid in the presence of a magnetic field was first studied by Yu and Shih [37]. They found that the magnetic field increases the stability of the fluid layer. El-Amin [38] examined the combined effect of internal heat generation and magnetic field on free convection and mass transfer flow in a micropolar fluid with constant suction. He proved that enhancement in heat transfer decreases with increase of magnetic field but it increases with increase of internal heat source strength. Later on Chamkha and Mudhaf [39] studied the MHD heat and mass transfer from a rotating vertical cone with heat generation or absorption effects. The extension for nanofluid case was also studied by Chamkha and Aly [40]. They showed that the presence of nanoparticles in the pure fluid had clear effects on the heat and mass transfer characteristics.

The main objective of this study is to investigate the effect of magnetic field on the onset of convection in a nanofluid layer induced by purely internal heating, such as that produced by microwave heating or chemical reaction, instead of bottom heating. Actually, the case of internal heating is mainly interesting, because not only the basic thermal gradient changes sign within the layer but also convection. Due to the presence of internal heating, the resulting generalized eigenvalue problem is found to be not possible for exact analytical treatment. Hence, the critical stability parameters are extracted numerically using the higher order Galerkin method for different types of thermal boundary conditions. Effects of magnetic field parameter, nanoparticle Rayleigh number, Lewis number, modified specific heat increment and modified diffusivity ratio on the onset of convection are examined for alumina-water nanofluid. Also the comparison of present results with the numerical results available in literature is given as a special case of the present study.

2. Problem formulation

We consider convection due to uniform internal heating of strength S_0^* in a system consisting of a horizontal layer of an incompressible electrically conducting nanofluid of thickness L. A Cartesian coordinate system (x, y, z) is chosen such that the origin is at the bottom of the layer and the gravity is acting in the negative vertical z-direction. A uniform vertical magnetic field $\tilde{\mathbf{H}}^* = (0, 0, H_0^*)$ acts on the system. The schematic diagram of the basic system is as shown in Fig. 1. The reference scale for the temperature and the volumetric fraction of nanoparticles are taken to be T_0^* and ϕ_0^*, respectively. Asterisks are used to distinguish the dimensional variables from the non-dimensional variables (without asterisks).

2.1. Assumptions

The mathematical equations describing the physical model are based upon the following assumptions:

(i) nanofluid is dilute and nanoparticles are suspended in the nanofluid using either surfactant or surface charge technology. Under this assumption, we can neglect the gravity segregation of nanoparticles in the governing equations;
(ii) the thermophysical properties except for density in the buoyancy force (Boussinesq hypothesis) are constant;
(iii) the fluid phase and nanoparticles are in thermal equilibrium state;
(iv) nanoparticles are spherical and not magnetic;
(v) nanofluid is incompressible, electrically conducting, Newtonian and laminar;
(vi) radiative heat transfer between the sides of wall is negligible when compared with other modes of the heat transfer.

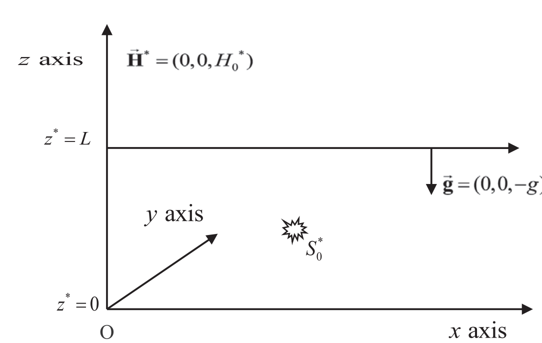

Fig. 1. Schematic diagram of the system considered here.