Computers and Fluids 121 (2015) 192–205

Contents lists available at ScienceDirect

Computers and Fluids

journal homepage: www.elsevier.com/locate/compfluid

A hybrid Taylor–Galerkin variational multi-scale stabilization method for the level set equation

Ahmed Bakkar [a,*], Wagdi G. Habashi [a,b], Marco Fossati [a], Guido S. Baruzzi [b]

[a] Department of Mechanical Engineering, CFD Laboratory, McGill University, 688 Sherbrooke Street West, Montreal, QC H3A 2S6, Canada
[b] Newmerical Technologies International, 680 Sherbrooke Street West, Montreal, QC H3A 2M7, Canada

ARTICLE INFO

Article history:
Received 13 June 2014
Revised 29 January 2015
Accepted 11 August 2015
Available online 18 August 2015

Keywords:
Level set
Variational multi-scale
Taylor Galerkin
Multiphase flow

ABSTRACT

A stabilized finite element formulation of the level set equation is proposed for the numerical simulation of water droplet dynamics for in-flight ice accretion problems. The variational multi-scale and Taylor–Galerkin approaches are coupled such that the temporal derivative in the weak Galerkin formulation is replaced with a Taylor series expansion improving the temporal accuracy of the scheme. The variational multi-scale approach is then applied to the semi-discrete equation, allowing the stabilization terms to appear naturally. Taylor series expansions up to the fourth order have been studied in terms of accuracy and convergence rates. A second order implicit expansion was found to provide a good trade-off between accuracy and computational cost when compared to a fourth order implicit expansion. Validation is done through a number of benchmark cases considering droplet stretching and high-speed advection. Results indicate good mass conservation characteristics compared to other methods available in the literature.

1. Introduction

Understanding and modeling the impingement dynamics of supercooled large droplets (SLDs) is crucial to be able to address accurately the mechanism of in-flight icing on aircraft and rotorcraft. This has become critical after the introduction of the new Appendix O for certification [1]. SLD are droplets of diameters greater than 50 μm which are found in the atmosphere in liquid state at temperatures below the freezing point as a consequence of the surface tension [1]. While small droplets are generally assumed to retain a nearly spherical shape even under substantial aerodynamic stresses, this is not true for SLD, especially at speeds typical of aerospace applications. In these conditions, droplets undergo great deformations and the phenomenology of the impact with aircraft surfaces is fairly complex. Aerodynamic distortion, break-up before impact, splashing and bouncing are phenomena that have to be taken into consideration when studying SLD dynamics. These phenomena could lead to water deposition over areas that would have remained dry in the case of the impact of smaller droplets. Failing to take the SLD behavior into account when designing ice protection systems may lead to the formation of ice on unprotected surfaces—a safety disaster.

The problem of SLD dynamics can be investigated either experimentally or numerically. Studying SLD experimentally in icing tunnels is not straightforward due to technical limitations that can affect the efficacy of the study. Atmospheric droplet distributions can be very difficult to reproduce, particularly those proposed in the new certification regulations. Replicating the flow conditions experienced in an in-flight icing scenario is also a challenging and expensive task, especially when scaling is involved [2]. Accurate surface impingement data are also nearly impossible to obtain in a non-invasive manner. Numerical methods, on the other hand, are capable of reproducing the flow conditions without the need for scaling and therefore can be considered as a valid instrument for this type of studies. A major challenge to numerical modeling of SLD is the ability to address the evolution of the surface of the droplets due to the aerodynamic shear that makes it highly irregular and prone to folding, pinching, merging and eventually break up. Large droplets impacting a surface at high speed can shatter and produce sprays of smaller droplets that re-enter the flow and deposit further downstream, by-passing the ice protection system [32,33]. Many reports can be found in the literature that propose numerical approaches to address the dynamics of droplets in in-flight icing. Nevertheless the majority of these methods make use of heuristic correlations based on single or multiple droplet experiments, or simplified analytical approaches that do not consider the SLD regime explicitly, or extrapolate the SLD behavior from non-SLD/non-aeronautical conditions and eventually may fail to provide accurate predictions [3,4]. Better insight in SLD dynamics could be obtained by considering numerical approaches to consistently study the detailed surface dynamics and evolution of a single and/or multiple SLD to remove some of the

* Corresponding author. Tel.: +15143981710.
E-mail address: ahmed.bakkar@cfdlab.mcgill.ca (A. Bakkar).

http://dx.doi.org/10.1016/j.compfluid.2015.08.008
0045-7930/

empiricism currently adopted in macroscopic analyses of droplet impact.

Single-droplet dynamics can be modeled with a Lagrangian, Eulerian, or mixed Eulerian–Lagrangian approach [5]. In a Lagrangian approach (also referred to as interface tracking), the grid coincides with the interface and tracking is explicitly accomplished by moving the grid points along with the interface. This method is generally accurate for cases involving small interface motion, but it is not effective when large deformations occur [5]. Lagrangian methods are usually computationally expensive due to the costs associated with a mesh that has to conform to the new interface location at every time step. In the mixed Eulerian–Lagrangian approach, the computations are done on a fixed Eulerian grid with marker points are used to define precisely the interface. These marker points are then used to guide and correct the interface location providing improved accuracy compared to a pure Eulerian approach. The major drawback of this approach is the difficulty associated with the tracking of the marker points when large deformations and topological changes occur, such as the merging of two droplets or the creation of non-simply connected regions [5]. The computational cost of mixed methods is reduced with respect to the Lagrangian approach, but still remains very high, especially when many markers are needed to accurately represent the interface. Lagrangian and mixed methods enjoy favorable properties when it comes to mass conservation since the tracking of the interface prevents any mass loss or gain, but their computational cost makes these methods unattractive. The Eulerian approach (also referred to as interface capturing) captures the interface on a fixed grid through a scalar function containing the interface information. The temporal and spatial evolution of the interface is updated by solving a transport equation. The Eulerian method offers: a greatly reduced computational cost and the possibility to define a consistent and unified numerical method that accounts simultaneously and accurately for the interaction between the different phases by resorting to mesh adaptation techniques. Despite the problems related to numerical stability and mass conservation, the Eulerian approach is recognized as a good candidate to address problems that involve the large deformation and topology changes of SLD dynamics.

The two most commonly adopted Eulerian methods are the level set method (LSM) [6,7] and the volume of fluid (VOF) method [8,9]. Both methods use the same advection equation but differ in the scalar function adopted for the advection problem. The LSM uses a signed distance function while the VOF method uses the volume fraction. The LSM, introduced by Osher and Sethian [10], provides an efficient and simple approach for modeling interface motion. Unlike the VOF method where the interface is reconstructed using volume fractions, the LSM provides a continuous interface representation allowing for the accurate account of the surface tension forces. Computing the surface normal and curvature is also straightforward due to the use of a signed distance function. These features make the LSM an attractive choice for modeling high-speed droplet impingement.

The LSM uses an implicit representation of the interface through the zero level set (LS) of a signed distance function. This function is evolved in time using a transport equation [6,11]. In the present work, a finite element (FE) implementation of the LSM was chosen due to its favorable numerical characteristics. These include the superior accuracy of the solution, the rigorous treatment of the boundary conditions, and the ability to handle anisotropic meshes with ease. The FE implementation of the LSM, however suffers from spurious oscillations for advection-dominated problems, which become even more noticeable in the case of the high speed flows typical of aeronautical applications. Stabilization techniques either introduce artificial diffusion, or an upwind discretization of the convection term. The addition of artificial diffusion offsets the negative diffusion introduced by the weak-Galerkin formulation, stabilizing the method, while the upwind formulation avoids the problems typical of the central difference approximation arising from the standard weak-Galerkin formulation [12]. Various FE implementations of the LSM have been published in the literature and some of the most relevant work will be outlined. Nagrath et al. [13] introduced a streamline-upwind/Petrov–Galerkin (SUPG) stabilized FE LSM for incompressible two-phase flows, which was used to model a bubble rising in a liquid. Valance et al. [14] developed a Galerkin least-squares stabilized Bubnov–Galerkin formulation for the LSM for applications involving irregular domains and discontinuities. Cho et al. [15] developed a direct re-initialization scheme for a Taylor–Galerkin (TG) stabilized FE-LSM for incompressible two-phase flows. The method was subsequently refined to include flows with surface tension [16]. Farthing and Kees [17] introduced two different stabilization methods for the LS equation. The first employed a Runge–Kutta Discontinuous Galerkin method. It was shown that, in some circumstances, this lead to entropy-violating solutions which are overcome by the addition of shock-capturing diffusion and viscous stabilization. The second approach employed a variational multi-scale (VMS) continuous Galerkin formulation, which was also found to provide inaccurate results for some test cases [17] and required the introduction of isentropic shock capturing terms. Kees et al. [18] later improved the mass conservation of the schemes by coupling them with the volume fraction equation.

In this paper, stabilization of the FE-LSM through VMS is further investigated. A hybrid TG-VMS approach is proposed in the attempt to enhance stabilization through the introduction of a Taylor series expansion for the temporal term. Temporal accuracy will also be improved with a higher-order temporal discretization, while the VMS approach will introduce the stabilization terms naturally. The outline of the paper is as follows: Section 2 introduces the equations to be modeled, Section 3 introduces the hybrid approach applied to the LS equation, Section 4 presents the numerical results and Section 5 contains the conclusions.

2. Mathematical model

In the LSM, the interface is represented by the zero LS of a scalar function φ. The signed distance function is the most commonly used and it is defined as the minimum distance between a grid node and the interface. The scalar distance function φ is then advected according to the following partial differential equation (PDE):

$$\varphi_t + \hat{u} \cdot \nabla\varphi = 0 \tag{1}$$

where $\hat{u}$ is the interface advection velocity, and φ is the signed distance function. By virtue of Eq. (1), the second order temporal derivative of the LS equation can be written as:

$$\varphi_{tt} = -u_t \cdot \nabla\varphi + (\hat{u} \cdot \nabla(\hat{u} \cdot \nabla\varphi)) \tag{2}$$

where $\hat{u}$ and u_t are, respectively, the average advection velocity and its first order temporal derivative, which are given by:

$$\hat{u} = \frac{u^{n+1} + u^n}{2} \quad \text{and} \quad u_t = \frac{u^{n+1} - u^n}{\Delta t} \tag{3}$$

where Δt is the time step used. The surface normal n and curvature k of the interface can be calculated using:

$$n = \frac{\nabla\varphi}{\|\nabla\varphi\|} \tag{4}$$

$$k = \nabla \cdot n = \nabla \cdot \frac{\nabla\varphi}{\|\nabla\varphi\|} \tag{5}$$

When using a signed distance function, the L2-norm of the gradient of the scalar function φ is equal to unity, simplifying Eqs. (4) and (5). However as Eq. (1) advects the interface with time, the function φ will drift from being a signed distance function and may become irregular [7]. This is because the velocity field will not necessarily transport all LSs with the same velocity [19]. This can lead to incorrect motion of the interface and may result in additional mass loss [20].