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Robust turbulent flow simulations using a Reynolds-stress-transport model on unstructured grids

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ABSTRACT

Progress toward a stable and efficient numerical treatment for the Reynolds-averaged Navier–Stokes equations with a Reynolds-stress-transport model on unstructured grids is presented. The unconditionally stable time marching scheme for Reynolds-stress-transport models, originally developed by the author for structured grids, is extended for unstructured grids using a finite volume method. The scheme guarantees the convergence of the fixed point iteration on the linearized problem. Moreover, the scheme is a positivity-preserving scheme, regardless of the time step. Thanks to the scheme characteristics, a spatially second-order discretization method for the Reynolds stress model equations (exactly as applied to the mean-flow equations) can be used, free of stability difficulties within the fixed point iterations. It is shown that the limiter has a dramatic influence on the convergence characteristics. Specifically, the limiter applied to the turbulence model variables was found to significantly influence the overall convergence behavior. Another key to the overall flow solver stability is a simple and robust procedure that is proposed to explicitly enforce all the realizability conditions of the Reynolds stress tensor. Two- and three-dimensional numerical flow simulations demonstrate the robustness of the overall flow solver for industrial applications.

1. Introduction

Since the early Reynolds averaged Navier–Stokes (RANS) turbulent flow simulations on unstructured grids, a first-order upwind approximation of the RANS turbulence model convective flux has been most widely used [1]. A few previous studies have successfully utilized a second- or third-order upwind biased approximation of the convective flux of RANS turbulence models [2] in structured grid-based flow solvers, and even of Reynolds stress transport models [3]. However, the use of a second-order, or a higher order accuracy for the approximation the turbulence model equations convective flux, in an unstructured grid-based flow solver is very rare [4]. The rationale behind the choice of a first order upwind scheme is to tackle the numerical stiffness of the turbulence model equations.

Surprisingly, it seems that the use of a first-order upwind scheme for convective flux of the RANS turbulence model equations is the most practical stabilization technique even though it is well known that the numerical stiffness originates from the turbulence model source terms. It is the best practice, especially when using unstructured grids, where the numerical difficulties are aggravated. That raises a question: why is a first order upwind scheme required to stabilize the numerical scheme even when the source term is treated appropriately ? An excellent analysis about the role of the convective flux in the stability of RANS turbulence models was conducted by Jongen and Marx [5]. Contrary to the classical understanding, they showed that the discretization of the convective flux is the source of most numerical difficulties. Specifically, it is the convective flux that may generate spurious oscillations, which may in turn result in a non physical solution. Namely, negative values of the model working variables, that are positive by virtue of the underlying physics, may appear. They have managed to isolate the origin of the problem and show that it is the divergence term (regardless of using a conservative or a non-conservative form) that is responsible for the numerical stability issues. Moreover, Jongen and Marx warn from the erroneous thought that the *deferred correction* approach, in which a combination of a first-order upwind approximation of the implicit term with a high-order TVD approximation of the residual (explicit) term, is TVD and hence a stable scheme can be obtained.

Generally, despite the mismatch between the explicit and implicit parts, the resulting time integration scheme, when applied to the mean flow equations, is very robust. However, this is not the case when a source term is involved, *e.g.*, when RANS

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turbulence model equations are considered. The convective flux is a linear operator with regards to the turbulence model working variables. However, for high-order schemes, certain nonlinearity is introduced into the discrete approximation (*e.g.*, a limiter in the widely used MUSCL schemes, or nonlinear weights in WENO schemes) in an attempt to control spurious oscillations in the solution. These nonlinear terms may vary significantly between neighboring cells due to the local nature of the turbulence model source term. This may especially be dramatic in the transition phase of the simulation through convergence where large errors and sharp interfaces between turbulent and non-turbulent regions exist. Therefore, the deferred correction approach may lead to a great mismatch between the explicit and implicit operators. Consequently, the deferred correction approach may results in severe numerical instability difficulties and most likely in a loss of positivity. This mismatch becomes even more severe when using unstructured grids. On structured grids, the stencil of the discrete convective flux involves cells that lie on a line which is perpendicular to the surface control volume face. For example, a third-order upwind biased scheme involves only three cells (on both sides of the face). While for an unstructured grid based flow solver, the discrete convective flux stencil involves more cells, hence a greater mismatch is present.

The present study focuses on designing a robust, implicit time marching scheme for the compressible Favre–Reynolds-averaged Navier–Stokes equations with RANS turbulence models on unstructured grids. Specifically, the motivation is to design a robust scheme for Reynolds stress transport models. Among RANS turbulence models, Reynolds-stress-transport models (RSTM), representing more elaborate physics, are perceived as the most advanced ones and are indeed regarded as the most stiff models.

To date, in most studies involving RSTM, the convergence that was presented was far from optimal. Moreover, the very few that presented convergence plots seldom meet the acceptable standards of convergence as with two-equation turbulence models. Chassain et al. [3] warrants that sometimes the insufficient convergence observed with RSTM is misinterpreted as a result of flow instability, when in fact it may be related to numerical instabilities. Therefore, there is a clear need for progress in developing convergent methods for RSTM, and by that to obtain reliable numerical solutions. Recently, significant progress was achieved in the design of a robust scheme for Reynolds stress transport models using a third-order upwind biased approximation on structured grids [6]. The scheme is based on the extension of the unconditionally positive-convergent implicit scheme (UPC), originally developed for two-equations turbulence models [7]. In contrast to the common deferred correction approach, the implicit operator within the UPC scheme is constructed directly from the discrete high-order explicit operator, namely there is no mismatch between the implicit and explicit operators. The present work describe the extension of the scheme presented in Ref. [6] to unstructured grids. Additionally, a stability analysis of the convective and diffusive fluxes of a RANS turbulence model is presented.

2. Governing equations

The governing equations are obtained by Favre–Reynolds averaging the Navier–Stokes equations (RANS) and modeling the Reynolds stress. The unknown, Reynolds stress tensor is modeled in this work via the *SSG/LRR-ω* Reynolds stress transport model developed by Eisfeld [8]. Note that the latest version of the model [9] is implemented herein. In the proceeding equations, the symbol $(\bar{\ })$ indicates non-weighted averaging, the symbol $(\tilde{\ })$ signifies Favre averaging, and the symbol $('')$ denotes Favre fluctuations.

2.1. Mean-flow equations

In a compact vector form, the mean-flow equations may be expressed in Cartesian coordinates as follows:

$$\frac{\partial \boldsymbol{Q}}{\partial \tau} + \frac{\partial(\boldsymbol{E}^c - \boldsymbol{E}^d)}{\partial x} + \frac{\partial(\boldsymbol{F}^c - \boldsymbol{F}^d)}{\partial y} + \frac{\partial(\boldsymbol{G}^c - \boldsymbol{G}^d)}{\partial z} = 0 \tag{1}$$

where τ denotes the time and x_i=[x, y, z] denote the Cartesian coordinates. The vector $\boldsymbol{Q}$ denotes the mean-flow dependent variables given as:

$$\boldsymbol{Q} = \left[\bar{\rho}, \bar{\rho}\tilde{u}, \bar{\rho}\tilde{v}, \bar{\rho}\tilde{w}, \tilde{E}\right]^T \tag{2}$$

The fluid density is denoted by ρ, the Cartesian velocity vector components are denoted by u, v, and w, and the total energy is denoted by E. The mean-flow inviscid fluxes are given by

$$\boldsymbol{E}^c = \begin{bmatrix} \bar{\rho}\tilde{u} \\ \bar{\rho}\tilde{u}\tilde{u} + \bar{p} \\ \bar{\rho}\tilde{v}\tilde{u} \\ \bar{\rho}\tilde{w}\tilde{u} \\ (\tilde{E} + \bar{p})\tilde{u} \end{bmatrix}, \quad \boldsymbol{F}^c = \begin{bmatrix} \bar{\rho}\tilde{v} \\ \bar{\rho}\tilde{u}\tilde{v} \\ \bar{\rho}\tilde{v}\tilde{v} + \bar{p} \\ \bar{\rho}\tilde{w}\tilde{v} \\ (\tilde{E} + \bar{p})\tilde{v} \end{bmatrix}, \quad \boldsymbol{G}^c = \begin{bmatrix} \bar{\rho}\tilde{w} \\ \bar{\rho}\tilde{u}\tilde{w} \\ \bar{\rho}\tilde{v}\tilde{w} \\ \bar{\rho}\tilde{w}\tilde{w} + \bar{p} \\ (\tilde{E} + \bar{p})\tilde{w} \end{bmatrix} \tag{3}$$

where p denotes the pressure. The mean-flow diffusive flux vectors are given by

$$\boldsymbol{E}^d = \begin{bmatrix} 0 \\ \bar{\tau}_{xx} - \bar{\rho}\widetilde{\Re}_{xx} \\ \bar{\tau}_{xy} - \bar{\rho}\widetilde{\Re}_{xy} \\ \bar{\tau}_{xz} - \bar{\rho}\widetilde{\Re}_{xz} \\ \beta_x \end{bmatrix}, \quad \boldsymbol{F}^d = \begin{bmatrix} 0 \\ \bar{\tau}_{xy} - \bar{\rho}\widetilde{\Re}_{xy} \\ \bar{\tau}_{yy} - \bar{\rho}\widetilde{\Re}_{yy} \\ \bar{\tau}_{yz} - \bar{\rho}\widetilde{\Re}_{yz} \\ \beta_y \end{bmatrix}, \quad \boldsymbol{G}^d = \begin{bmatrix} 0 \\ \bar{\tau}_{xz} - \bar{\rho}\widetilde{\Re}_{xz} \\ \bar{\tau}_{yz} - \bar{\rho}\widetilde{\Re}_{yz} \\ \bar{\tau}_{zz} - \bar{\rho}\widetilde{\Re}_{zz} \\ \beta_z \end{bmatrix} \tag{4}$$

where $\bar{\tau}_{x_i x_j}$ and $\widetilde{\Re}_{x_i x_j} = \widetilde{u_i'' u_j''}$ are the components of the viscous stress and Reynolds-stress tensors, respectively. The terms β_{x_i} are given by:

$$\begin{aligned}\beta_{x_i} &= \tilde{u}(\bar{\tau}_{x_i x} - \bar{\rho}\widetilde{\Re}_{x_i x}) + \tilde{v}(\bar{\tau}_{x_i y} - \bar{\rho}\widetilde{\Re}_{x_i y}) \\ &\quad + \tilde{w}(\bar{\tau}_{x_i z} - \bar{\rho}\widetilde{\Re}_{x_i z}) - \bar{q}_{x_i} - (\bar{q}_t)_{x_i}\end{aligned} \tag{5}$$

where $\bar{q}_{x_i}$ and $(\bar{q}_t)_{x_i}$ are the molecular and turbulent heat flux, respectively, modeled using Fourier's law:

$$\bar{q}_{x_i} = -\bar{\kappa}\bar{T}_{,x_i} \tag{6}$$

$$(\bar{q}_t)_{x_i} = -\bar{\kappa}_t\bar{T}_{,x_i} \tag{7}$$

with T denoting the temperature and $\bar{\kappa} = c_p\bar{\mu}/Pr$ and $\bar{\kappa}_t = c_p\bar{\mu}_t Pr_t$ are the molecular and turbulent heat conductivities, respectively. The term $\bar{\mu}$ denotes the molecular viscosity, calculated using Sutherland's law. The term μ_t denotes the turbulent viscosity, whereas for the *SSG/LRR-ω* RSTM it is calculated as $\mu_t = \rho k/\omega$. With $k = \frac{1}{2}\widetilde{\Re}_{x_i x_i}$ being the turbulent kinetic energy, and ω is the specific turbulent dissipation rate. The term c_p is the specific heat capacity at constant pressure while $Pr = 0.72$ and $Pr_t = 0.9$ are the molecular and turbulent Prandtl numbers, respectively. The mean-flow equations are closed using the equation of state for a perfect gas, given by:

$$\bar{p} = (\gamma - 1)\left[\tilde{E} - \frac{1}{2}\bar{\rho}\left(\tilde{u}^2 + \tilde{v}^2 + \tilde{w}^2\right)\right] \tag{8}$$

where γ is the ratio of specific heats (c_p/c_v), set to γ=1.4. Note that the contribution of the turbulent diffusion to the total energy transport equation is neglected, as well as the contribution of the turbulent kinetic energy to the total energy.