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On spatial filtering of flow variables in high-order finite volume methods

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ABSTRACT

A new method of spatial filtering in high-order finite volume methods is presented and assessed. The base of this method is to filter face-averaged variables (fluxes) and then the recovery of cell-averaged ones. Two kinds of filtering method are proposed. The first kind is highly dissipative and appropriate for the numerical regions that need high dissipation, e.g. sponge zones. The second kind, on the other hand, is a precise method and hence is suitable for applying the high-order finite difference filters to the finite volume methods. Applying high-order finite difference filters directly to the high-order finite volume methods without using the proposed method causes stability problems in the large eddy simulation of high Reynolds number flows. The test cases, namely, propagation of the one-dimensional wave packet, advection of a two-dimensional vortical wave and the large eddy simulation of turbulent round jet at $Re = 10^4$ and 10^5 are used to examine the accuracy and performance of the proposed methods. The last test case shows the effectiveness of the proposed methods in the large eddy simulation of flows by high-order FVMs. The performance of filtering methods on a curvilinear wavy grid is also investigated.

1. Introduction

Spatial filtering is frequently used in the high-order and high-resolution numerical methods aiming at computational aeroacoustics (CAA), direct numerical simulation (DNS), and large eddy simulation (LES). The high wave number oscillations associated to the scales near the grid size can lead to the stability problem in such simulations. These waves generated from the physical or numerical sources are then removed by applying the numerical filters [1–6]. These filters are moreover suitable for explicit filtering of the flow variables in the large eddy simulation [7–13] and regularization modeling [14,15] of turbulent flows. The governing equations of such a simulation do not include the scales finer than the grid size and therefore these scales should be removed from the numerical solution at each time step. Primitive large eddy simulations are performed without any explicit filter by supposing that the finite computational mesh and discrete derivative operators have low-pass filtering effects. However, since this inherent filter acts only in one spatial direction in which the derivative is taken and it has uncontrolled dissipation [11], the later simulations use explicit filtering by applying the known filters. These issues are the main reasons of high-order filter development efforts in the recent decade [1–5]. Nearly all of these filters are developed to use in the finite difference methods which impose some restrictions for applying them in the finite volume methods. For example, the first and last values of variable vector are not included in the formulations of these filters by supposing that they are determined from the boundary conditions. The present work introduces a procedure for filtering of cell-averaged variables in the FVMs. The base of this procedure is to filter face-averaged variables (fluxes) and then the recovery of cell-averaged ones. A new compact formulation is proposed here for the recovery step. This formulation besides an older compact formulation results in two kinds of filtering method. The first kind is highly dissipative and appropriate for the numerical regions that need high dissipation, e.g. sponge zones. The second kind, on the other hand, is a precise method and hence is suitable for applying the high-order finite difference filters to the FVMs. In the next sections, the presented filtering method is described and assessed by some numerical test cases, including advection of one dimensional wave packet, convection of two-dimensional vortical wave and the large eddy simulation of round jet at $Re = 10^4$ and 10^5.

2. Filtering method

Consider a two-dimensional grid with numbering as shown in Fig. 1. The cell- and face-averaged parameters on such a grid are

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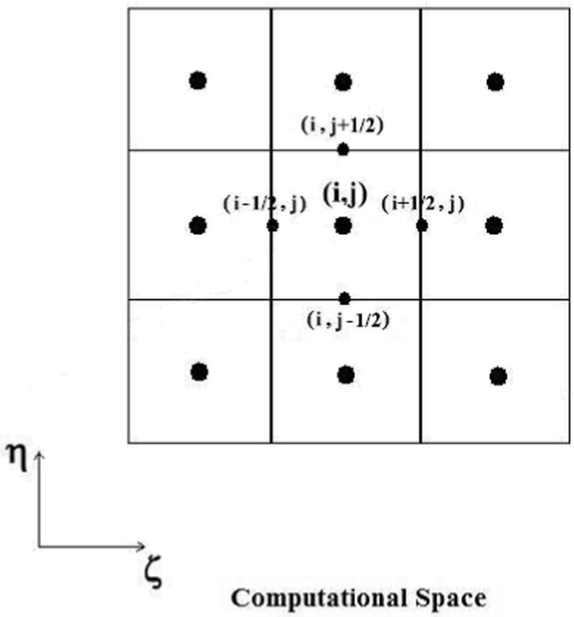

Fig. 1. A tow-dimensional grid in the computational space.

defined, respectively, as follows [16–18]:

$$\overline{\phi}^{\xi\eta}_{i,j} = \frac{1}{\Delta\xi\Delta\eta}\int_{\xi_{i-\frac{1}{2}}}^{\xi_{i+\frac{1}{2}}}\int_{\eta_{j-\frac{1}{2}}}^{\eta_{j+\frac{1}{2}}} \phi(\xi,\eta)\,d\xi d\eta \quad (1)$$

$$\overline{\phi}^{\eta}_{i+\frac{1}{2},j} = \frac{1}{\Delta\eta}\int_{\eta_{j-\frac{1}{2}}}^{\eta_{j+\frac{1}{2}}} \phi(\xi_{i+\frac{1}{2}},\eta)\,d\eta \quad (2)$$

where ξ and η are the computational space coordinates. In the finite volume methods the face-averaged fluxes are obtained from the cell-averaged variables. For example, these fluxes are computed by the following formulation in the compact FVM [16–18]:

$$a\overline{\phi}^{\eta}_{i-\frac{1}{2},j} + \overline{\phi}^{\eta}_{i+\frac{1}{2},j} + a\overline{\phi}^{\eta}_{i+\frac{3}{2},j} = b\left(\overline{\phi}^{\xi\eta}_{i,j} + \overline{\phi}^{\xi\eta}_{i+1,j}\right) \quad (3)$$

where, to have a fourth-order accuracy, the constants a and b are obtained as follows [16]:

$$a = \frac{1}{4},\ b = \frac{3}{4} \quad (4)$$

Knowing the fluxes, the cell-averaged variables advance in time according to the governing equations. These cell-averaged values are then filtered by the proposed method. Since the fluxes are available in the numerical process of FVM, they are used as a medium of filtering in the proposed method. So, the computation of the fluxes by means of Eq. (3) is the first step of filtering. The next step is to filter these fluxes by the finite difference filters. It must be noted that this step can be performed without any restriction. The filtered cell-averaged values are then recovered from these filtered fluxes. This recovery process needs a formulation similar to Eq. (3). Here, to satisfy this requirement a compact formulation is introduced as follows:

$$c\overline{\phi}^{\xi\eta}_{i-1,j} + \overline{\phi}^{\xi\eta}_{i,j} + c\overline{\phi}^{\xi\eta}_{i+1,j} = d\left(\overline{\phi}^{\eta}_{i-\frac{1}{2},j} + \overline{\phi}^{\eta}_{i+\frac{1}{2},j}\right) \quad (5)$$

The coefficients in this equation are obtained by matching Taylor's expansion on both the sides up to the fourth-order. The two-dimensional Taylor series of parameter ϕ up to the fourth-order is as follows:

$$\phi = \phi_{i,j} + \phi_\xi\xi + \phi_\eta\eta + \phi_{\xi\eta}\xi\eta + \frac{1}{2}\phi_{\xi\xi}\xi^2 + \frac{1}{2}\phi_{\eta\eta}\eta^2 + \frac{1}{2}\phi_{\xi\xi\eta}\xi^2\eta + \frac{1}{2}\phi_{\xi\eta\eta}\xi\eta^2 + \frac{1}{6}\phi_{\xi\xi\xi}\xi^3 + \frac{1}{6}\phi_{\eta\eta\eta}\eta^3 \quad (6)$$

Then the terms involved in Eq. (5) are obtained by using Eqs. (1) and (2) as follows:

$$\begin{aligned}
\overline{\phi}^{\xi\eta}_{i-1,j} &= \phi_{i,j} - \phi_\xi\xi + \frac{13}{24}\phi_{\xi\xi}\xi^2 + \frac{1}{24}\phi_{\eta\eta}\eta^2 - \frac{1}{24}\phi_{\xi\eta\eta}\xi\eta^2 - \frac{5}{24}\phi_{\xi\xi\xi}\xi^3 \\
\overline{\phi}^{\xi\eta}_{i,j} &= \phi_{i,j} + \frac{1}{24}\phi_{\xi\xi}\xi^2 + \frac{1}{24}\phi_{\eta\eta}\eta^2 \\
\overline{\phi}^{\xi\eta}_{i+1,j} &= \phi_{i,j} + \phi_\xi\xi + \frac{13}{24}\phi_{\xi\xi}\xi^2 + \frac{1}{24}\phi_{\eta\eta}\eta^2 + \frac{1}{24}\phi_{\xi\eta\eta}\xi\eta^2 + \frac{5}{24}\phi_{\xi\xi\xi}\xi^3 \\
\overline{\phi}^{\eta}_{i-\frac{1}{2},j} &= \phi_{i,j} - \frac{1}{2}\phi_\xi\xi + \frac{1}{8}\phi_{\xi\xi}\xi^2 + \frac{1}{24}\phi_{\eta\eta}\eta^2 - \frac{1}{48}\phi_{\xi\eta\eta}\xi\eta^2 - \frac{1}{48}\phi_{\xi\xi\xi}\xi^3 \\
\overline{\phi}^{\eta}_{i+\frac{1}{2},j} &= \phi_{i,j} + \frac{1}{2}\phi_\xi\xi + \frac{1}{8}\phi_{\xi\xi}\xi^2 + \frac{1}{24}\phi_{\eta\eta}\eta^2 + \frac{1}{48}\phi_{\xi\eta\eta}\xi\eta^2 + \frac{1}{48}\phi_{\xi\xi\xi}\xi^3
\end{aligned} \quad (7)$$

By substituting these series in Eq. (5), the coefficients are obtained as follows:

$$c = \frac{1}{10},\ d = \frac{3}{5} \quad (8)$$

Now, two different methods can be used for recovering the filtered cell-averaged parameters from the filtered face-averaged ones by means of Eqs. (3) and (5). The first method labeled as "Method 1" uses Eq. (5) for the recovery process. However, Eq. (5) cannot be applied to the end cells for a non-periodic boundary condition. Eq. (3) is therefore proposed for using in these cells and completing the related tri-diagonal matrix. The second method labeled as "Method 2" uses Eq. (3) for the recovery of cell-averaged values from the filtered face-averaged ones. When this equation is applied in a line with n cells, $n - 1$ equations are obtained. One additional equation needed to complete the system of equations is obtained from applying Eq. (5) to the last but one cell. The resulted tri-diagonal matrix in both the methods is solved by the Thomas algorithm. Using the fourth-order formulations in all cells, including the interior and the end ones, both the methods have a fourth-order accuracy. However, since a same equation is used for computing fluxes from the cell-averaged values and the inverse computation in Method 2, the order of accuracy of this method depends only on the order of filter used for the fluxes. For the multi-dimensional problems the mentioned filtering process can be applied sequentially in each direction.

Filtering of the face-averaged values which is the second step of both the presented methods is made by the formerly presented explicit or implicit filters. Explicit filters have the following general formulation:

$$\hat{\phi}_i = \phi_i - \sigma_d\sum_{n=-p}^{q} d_n\phi_{i+n} \quad (9)$$

where the symbol "^" represents the filtered value and σ_d is the filtering strength taken between 0 and 1. Higher values of σ_d in this range represent a more dissipative filter. The central formulation of this filter for the interior points is presented by $p = q$ [2]. The coefficients d_n are classically obtained by canceling the π-mode and equating Taylor's expansion of both sides of Eq. (9) up to the desired order of filter [19]. However, some works [1,2] have obtained these coefficients by optimizing the transfer function of the filter to minimize the dissipation of filtering in a desirable range