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# Direct numerical simulation of aeroacoustic sound by volume penalization method

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ABSTRACT

The volume penalization (VP) method for compressible flows is investigated as a tool of direct numerical simulation of aeroacoustic sound in problems where not only acoustic pressure but also hydrodynamic pressure depends on time and position. First, it is shown that the method proposed by Liu and Vasilyev (2007) [30] is not Galilean invariant. It is corrected to satisfy Galilean invariance. Next, numerical accuracy of the corrected VP method is investigated in problems of simple geometry which can be simulated also by a standard method on a body-fitted coordinate system: sound generation in (i) flow past a fixed square/circular cylinder, (ii) flow past an oscillating square/circular cylinder, and (iii) flow past two square cylinders. The results confirm that the corrected VP method gives reasonably accurate results for sound pressure which is much smaller than hydrodynamic pressure within 5% error. Finally, the corrected method is applied to two examples of complex geometry, which cannot be simulated by standard methods using body-fitted coordinate systems without considerable difficulty: sound generation in (i) flow past an oscillating cylinder and a fixed cylinder behind it and (ii) flow past a bundle of cylinders. The results show that the present method is in principle applicable to aeroacoustic problems in any complex geometry including practical engineering ones.



## 1. Introduction

The aeroacoustic noises radiated from airplanes, high-speed trains, helicopters, wind turbines etc. are one of the great concerns these days since they have significant negative effects on our daily lives. The problem is becoming more and more serious as the speed of the trains or the wings increases since the power of the noises increases significantly as $M^5 \sim M^8$, where $M$ is the Mach number based on the characteristic flow velocity and the exponent depends on the problem. Thus there is increasing need for reduction of the aeroacoustic noises in a variety of industrial applications.

In order to reduce the aeroacoustic noises we should understand the mechanism of their generation and propagation. A number of experimental efforts have been devoted for it. However, there are a few difficulties in investigating aeroacoustic noises in the experiments: it is difficult to remove background noises which contaminate the aeroacoustic noises; it is also difficult to obtain field data of physical variables including pressure and velocity, although recent progress in time-resolved particle image velocimetry is worth noting [1,2]. Theoretical studies have been successful since the pioneering work by Lighthill [3]. Various types of theory or acoustic analogy are available. In order to predict the aeroacoustic noises time-varying field data of sound sources should be provided; this is done by numerical simulation in one branch of computational aeroacoustics called a hybrid method. In the hybrid method, however, the accuracy depends on to what extent the assumptions made in the theories are satisfied.

Direct numerical simulation (DNS) of the aeroacoustic sound emerged as another branch of computational aeroacoustics about two decades ago. Finite difference methods with high accuracy and non-reflecting boundary conditions, together with rapid growth of computer power, are combined to overcome three difficulties: (i) the sound pressure of the aeroacoustic noises is usually much smaller than the ambient pressure; (ii) acoustic waves are reflected at the far boundaries of the computational domain when conventional boundary conditions are used; and (iii) a large number of grid points are required to cover both the flow and the sound regions. See Colonius and Lele [4] and Wang et al. [5] for computational aeroacoustics. A number of aeroacoustic problems

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have been successfully solved including the sound radiated by co-rotating vortices [6], sound generation in a mixing layer [7], a jet [8,9], and a cavity flow [10], sound generation by collision of vortex rings [11,12] and unsteady motion of a cylinder [13], and the Aeolian tones [14–16]. However, these studies are limited to flows in a simple geometry since most of the finite difference methods with high accuracy cannot be applied to complex geometries where unstructured mesh systems are used. This limitation implies that we cannot simulate directly the aeroacoustic noises from high-speed trains, helicopters, and wind turbines, all of which have complex geometries in the sense that the boundaries of rigid bodies have different length scales and various directions. The geometry can be even deformable as the rotor of a wind turbine rotates while the main rotor shaft is fixed generating low-frequency noises due to their interaction; it is challenging to capture the noises by existing methods of DNS since a body-fitted coordinate (BFC) system should deform too or an overset grid system should be implemented without losing high accuracy. Thus methods for DNS of aeroacoustic sound which can be used for complex and/or deformable geometries are expected.

One possible way to realize DNS of aeroacoustic sound in complex geometries is to use the immersed boundary methods, which are widely used for flows in complex geometries [17]. There are several studies in which the immersed boundary method is used for compressible flow. It is Chung and Morris [18] who applied the immersed boundary method in acoustic scattering problem for the first time. Chaudhuri et al. [19] studied shock/obstacle interactions. Acoustic scattering, which is essentially a linear problem, is studied by a few groups [20–22]. To our knowledge, however, there has been no DNS of the aeroacoustic sound, which is a nonlinear problem, by the immersed boundary method although Seo and Mittal [20] studied the aeroacoustic sound by a hybrid method.

The volume penalization (VP) method, which is one of the immersed boundary methods, has been applied mostly to incompressible flows [23–28]. One of the advantages of the VP method for incompressible flows is that it has a firm mathematical basis. In the VP method we solve the Navier–Stokes equations supplemented by penalization terms instead of imposing no-slip boundary conditions at the surface of rigid bodies in the flow. The solutions to this penalized Navier–Stokes equations converge to those to the original problem of the Navier–Stokes equations with no-slip boundary conditions in the limit of vanishing permeability [23,29]. For compressible flows, on the other hand, there are only a few studies. Liu and Vasilyev [30] introduced a new penalization term into the mass conservation law and showed that reflection of acoustic waves is correctly captured by it. Boiron et al. [31] studied shock/obstacle interaction by the VP method. Recently Brown-Dymkoski et al. [32] proposed a method to implement the Neumann and the Robin boundary conditions in the VP method in addition to the Dirichlet boundary conditions. Mathematical results are known for a restricted case of homentropic flows [33].

In this paper we study the applicability of the VP method to DNS of the aeroacoustic sound. Our aim is to obtain acoustic waves and flow fields simultaneously with sufficient accuracy. It should be emphasized that there has been no such study, while previous studies have focused on the accuracy of only flow fields or acoustic waves in the absence of flow fields. This is not a simple problem since the pressure cannot be decomposed into acoustic pressure and hydrodynamic pressure by a simple method. In fact, in low Mach number flows in which the aeroacoustic sound source is compact the acoustic pressure is usually proportional to $M^{2.5\sim4}r^{-1/2}$ and $M^{3\sim4}r^{-1}$ in two dimensions and three dimensions, respectively, where $r$ is the distance between the observation point and the sound source region, while the hydrodynamic pressure is proportional to $M^2r^{-2}$ and $M^2r^{-3}$, respectively. Thus the acoustic pressure which is dominant in the far field is often undetectable by existing methods since it is much smaller than the hydrodynamic pressure and is the same order as numerical error in the flow region. More importantly, we will show that the method proposed by Liu and Vasilyev [30] should be corrected to satisfy Galilean invariance.

The paper is organized as follows. In Section 2 we show that the method proposed by Liu and Vasilyev [30] is not Galilean invariant and a corrected method is presented. After describing the numerical methods in Section 3, numerical accuracy of the corrected method is investigated in Section 4. We choose a simple geometry so that the aeroacoustic sound is calculated both by the VP method and by a standard method. Comparison between the results by the two methods should provide a good validation of the VP method since it has been well established that the standard method gives accurate results. We consider the sound generated in a flow past a fixed square cylinder (Section 4.1), a flow past an oscillating square cylinder (Section 4.2), a flow past two square cylinders in a side-by-side arrangement (Section 4.3), and a flow past a fixed/oscillating circular cylinder (Section 4.4). Then we show an example of application to deformable geometries in Section 5. The final section Section 6 concludes the paper.

## 2. Volume penalization method for compressible flow

### 2.1. Method proposed by Liu and Vasilyev [30]

In the VP method for compressible flow proposed by Liu and Vasilyev [30] the compressible Navier–Stokes equations are complemented by penalization terms as follows

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x_j}(\rho u_j) = -\left(\frac{1}{\phi} - 1\right)\chi \frac{\partial}{\partial x_j}(\rho u_j), \tag{1}$$

$$\frac{\partial \rho u_i}{\partial t} + \frac{\partial}{\partial x_j}\left(\rho u_i u_j\right) = -\frac{\partial p}{\partial x_i} + \frac{\partial \tau_{ij}}{\partial x_j} - \frac{\chi}{\eta}(u_i - U_{0,i}), \tag{2}$$

$$\frac{\partial e}{\partial t} + \frac{\partial}{\partial x_j}[(e+p)u_j] = \frac{\partial}{\partial x_j}(u_i \tau_{ij}) + \frac{\partial}{\partial x_j}\left(\kappa \frac{\partial T}{\partial x_j}\right) - \frac{\chi}{\eta_T}(T - T_0), \tag{3}$$

where $\rho$ is the density of the fluid, $u_i$ is the velocity, $p$ is the pressure, $\tau_{ij} = \mu(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} - \frac{2}{3}\frac{\partial u_l}{\partial x_l}\delta_{ij})$ is the viscous stress tensor, $e$ is the total energy, $T$ is the temperature, $U_{0,\,i}$ and $T_0$ are the velocity and the temperature of the rigid bodies, respectively, $\phi$ is the porosity, $\eta$ is the viscous permeability, and $\eta_T$ is the thermal permeability. Usually the permeabilities are assumed small: $0 < \eta, \eta_T \ll 1$. Throughout the paper the viscosity $\mu$ and the thermal conductivity $\kappa$ are assumed to be constant. The Prandtl number $Pr = \gamma\mu/\kappa$ is set to 0.72, where the ratio of the specific heats $\gamma$ is set to 1.4. The fluid is assumed to be an ideal gas; the equation of state

$$p = \rho RT = (\gamma - 1)\left(e - \frac{1}{2}\rho u_i u_i\right), \tag{4}$$

where $R$ is the gas constant, closes the set of equations.

In the VP method the boundaries between the fluid and the rigid bodies need not coincide with surfaces of grid points. Instead the mask function $\chi$ defined by

$$\chi(\boldsymbol{x}, t) = \begin{cases} 1 & \text{if } \boldsymbol{x} \in \text{rigid bodies}, \\ 0 & \text{otherwise} \end{cases} \tag{5}$$

differentiates the fluid and the rigid bodies in Eqs. (1)–(3). The terms which involve $\chi$ are the penalization terms. In the flow region, where $\chi = 0$, the equations reduce to the ordinary compressible Navier–Stokes equations. On the other hand, in the rigid