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A novel Hermite Taylor Least Square based meshfree framework with adaptive upwind scheme for two dimensional incompressible flows

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ABSTRACT

A new meshfree framework based on Hermite Taylor Least Square Finite Difference method is proposed. The conventionally used Least Square Finite Difference (LSFD) scheme with ghost point method for Neumann boundary conditions is known to have shortcomings especially for irregular nodal distributions. In this work, the performance of the LSFD scheme is augmented by incorporating a novel Hermite Taylor Least Square (HTLS) method for easy and efficient implementation of the Neumann boundary conditions. The method is initially validated by solving a Poisson equation with both Dirichlet and Neumann boundary conditions. With its promising numerical performance, the method is extended to the full Navier–Stokes equations in two dimensions. An innovative adaptive upwind scheme is adopted to handle the convective terms in the momentum equations by modifying the support domain in the upstream direction. By using a modified Euclidean distance function according to the local flow direction and the value of parameter that controls the convection effect (mesh Peclet number), the local support domain can be shifted towards the upstream direction thereby naturally incorporating the upwind effect while computing the coefficients for the LSFD method. The Navier–Stokes equations are solved in a primitive variables (velocity and pressure) approach by using a first order semi-implicit projection method. In order to validate the developed framework, three flow problems (lid driven cavity, channel flow and flow over a circular cylinder) are considered. All of these problems are well documented because of their benchmarking relevance. It is observed that the new framework produces results which match qualitatively as well as quantitatively with earlier established theory and observations and hence demonstrate its ability to successfully simulate flows of practical interest in an entirely meshfree approach.

1. Introduction

Although the conventional CFD methods like finite difference method (FDM), finite element method (FEM) and finite volume method (FVM) are highly matured disciplines and produces accurate and stable results in majority of the cases, the common bottleneck is the generation of a mesh. In the last decade, extensive research has been therefore conducted in the field of meshfree methods. These meshfree methods offer an alternative by circumventing the problems of conventional CFD methods either fully or partially. Instead of relying on elements or mesh, the meshfree methods use only the nodal coordinate information without the associated connectivity.

A number of meshfree methods are available in literature. These include the smooth particle hydrodynamics (SPH) [1], reproducing kernel particle method (RPKM) [2], element-free Galerkin (EFG) method [3], the local Petrov–Galerkin method [4], radial basic function (RBF) method [5,6], finite pointset method etc. In the present work, the Least Square Finite Difference (LSFD) is considered which was originally developed by Ding et al. [7]. The Least Square Finite Difference (LSFD), which is derived from the multi-dimensional Taylor series expansion, provides a more general discretization method for multi-dimensional geometries.

2. Development of the Least Square Finite Difference method (LSFD)

The LSFD method is based on the use of a weighted least square approximation procedure together with a Taylor series expansion of the unknown function. Multi-dimensional Taylor series expansion can be employed to approximate the unknown function within a local support of reference node.

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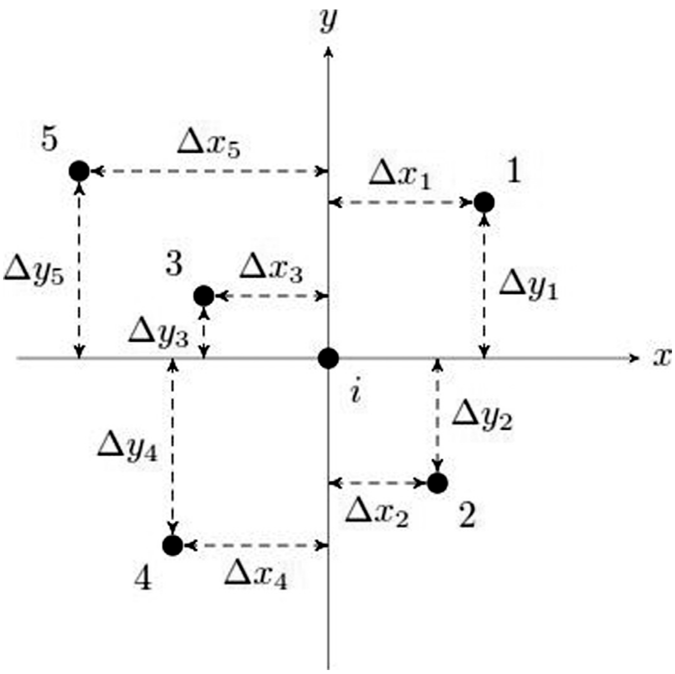

Fig. 1. Two dimensional node distribution [8].

For approximating derivatives up to second order, the truncated Taylor series expansion about i can be expressed as

$$f_j = f_i + \Delta x_j \frac{\partial f_i}{\partial x} + \Delta y_j \frac{\partial f_i}{\partial y} + \frac{\Delta {x_j}^2}{2}\frac{\partial^2 f_i}{\partial x^2} + \Delta x_j \Delta y_j \frac{\partial^2 f_i}{\partial x \partial y} + \frac{\Delta {y_j}^2}{2}\frac{\partial^2 f_i}{\partial y^2} + O(\Delta {x_j}^3, \Delta {y_j}^3) \tag{1}$$

where $j \in \{1,2,\ldots,N\}$ are the neighbor nodes of i and $(\Delta x_j, \Delta y_j)$ denotes the distance from the reference node i to neighbor node j as shown in Fig. 1.

The derivative at point i is determined by minimizing the sum of all the squared residuals i.e. Euclidean norm of error vector for all neighboring points of i under consideration known as support points) with respect to the five derivative terms defined as

$$\sum_{j=1}^{N} E_j^2 = \sum_{j=1}^{N} w_j \left[f_j - f_i - \Delta x_j \frac{\partial f_i}{\partial x} - \Delta y_j \frac{\partial f_i}{\partial y} - \frac{\Delta {x_j}^2}{2}\frac{\partial^2 f_i}{\partial x^2} - \Delta x_j \Delta y_j \frac{\partial^2 f_i}{\partial x \partial y} - \frac{\Delta {y_j}^2}{2}\frac{\partial^2 f_i}{\partial y^2} \right]^2 \tag{2}$$

The square error in Eq. (2) is well-known [9] to be minimized under the condition

$$S^T W \Delta f = (S^T W S) df \tag{3}$$

where Δf is a $N \times 1$ array, N being the number of neighbor nodes.

$$\Delta f = [f_j - f_i] \tag{4}$$

S is the $N \times P$ matrix

$$S = \left[\Delta x_j \Delta y_j \frac{\Delta {x_j}^2}{2} \Delta x_j \Delta y_j \frac{\Delta {y_j}^2}{2} \right] \tag{5}$$

and D is the $P \times 1$ array, P being the number of derivative terms.

$$df = \left[\frac{\partial f_i}{\partial x} \frac{\partial f_i}{\partial y} \frac{\partial^2 f_i}{\partial x^2} \frac{\partial^2 f_i}{\partial x \partial y} \frac{\partial^2 f_i}{\partial y^2} \right]^T \tag{6}$$

and W is the diagonal weight matrix.

$$W = diag[w_1 w_2 \ldots . w_{N-1} w_N] \tag{7}$$

Many weight functions are available in literature but according to [7], the following function provides slightly better accuracy than other commonly used weight functions and is hence applied in this work:

$$w_j = \sqrt{\frac{4}{\pi}}(1 - r_j^2)^4 \tag{8}$$

r_j is the distance of the neighbor node from reference node i.

Thus, the derivative array is given by

$$df = C \Delta f \tag{9}$$

where

$$C = (S^T W S)^{-1} S^T W \tag{10}$$

C is a geometric matrix depending only on the nodal coordinates and can be calculated during the preprocessor stage.

3. Implementation considerations

3.1. Neighbor node selection

As mentioned in the previous section, the LSFD method approximates the derivatives at a node by considering Taylor series expansion on a set of neighbor or supporting nodes. Therefore, the selection of the neighbor nodes is crucial for the performance of the method. There exist different criteria for the selection of neighbor nodes in literature. In this work, a nearest neighbor search will be applied. Each local subdomain will contain a pre-specified amount of nearest neighbors instead of selecting domains of constant radius. The nearest neighbor search criterion will ensure that the local subdomains in regions of higher nodal density will encompass a smaller area than that of a coarse nodal distribution. The objective is therefore, for every single node in the domain, to find its adjacent nodes in terms of Euclidean distance.

3.2. Neumann boundary condition enforcement

A popular method that has been used by several authors [10–12] to enforce a Neumann boundary condition has been the use of ghost points. However this method produces inaccurate results especially for irregular nodal distribution [13]. Due to this shortcoming, a new method called the Hermite Taylor Least Square (HTLS) method is used.

When a support domain contains nodes that belong to a boundary with a specified Neumann boundary condition, a specialized form of the Taylor series least squares method can be formulated [13]. The new formulation takes advantage of the fact that both function values and function derivatives are known at the boundary nodes. Including this additional data results in a more accurate approximation of derivatives at the reference node.

If there is a subset of N_b Neumann boundary nodes within the N supporting nodes of the support domain for reference node i, the truncated Taylor series for the first x and y derivatives at (x_j, y_j), $j \in \{1,2,\ldots,N_b\}$ can be expressed as

$$\frac{\partial f_j}{\partial x} = \frac{\partial f_i}{\partial x} + \Delta x_j \frac{\partial^2 f_i}{\partial x^2} + \Delta y_j \frac{\partial^2 f_i}{\partial x \partial y} + \cdots \tag{11}$$

$$\frac{\partial f_j}{\partial y} = \frac{\partial f_i}{\partial y} + \Delta x_j \frac{\partial^2 f_i}{\partial x \partial y} + \Delta y_j \frac{\partial^2 f_i}{\partial y^2} + \cdots \tag{12}$$

The gradient of the potential in the normal direction $\nabla f_j \cdot n_j$ can be expressed in terms of the Taylor series by taking the dot