Computers and Fluids 130 (2016) 94–108

Contents lists available at ScienceDirect

Computers and Fluids

journal homepage: www.elsevier.com/locate/compfluid

Numerical simulation of cross-flow around three equal diameter cylinders in an equilateral-triangular configuration at low Reynolds numbers

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ARTICLE INFO

Article history:
Received 9 June 2014
Revised 19 October 2015
Accepted 11 February 2016
Available online 26 February 2016

Keywords:
Cross-flow around three cylinders
Finite volume method
Numerical simulation
Unstructured mesh
Fluctuating force

ABSTRACT

Successful numerical simulation can reveal important flow characteristics and information which are extremely difficult to obtain experimentally. A two-dimensional finite volume method with unstructured mesh is used to simulate cross-flow around three equal diameter cylinders arranged in an equilateral-triangular interact manner for two incidence angles, $\alpha = 0^\circ$ and $\alpha = 180^\circ$. Special attention is paid on the variation of flow pattern characteristics among the cylinders, force characteristics and Strouhal number of each cylinder with nine spacing ratios (L/D) ranging from 1.5 to 7 at Re = 100 and 200, respectively. Three distinct flow patterns are revealed: (1) At $\alpha = 0^\circ$ (inverted-T shaped arrangement), the biased flow generated behind the side by side downstream cylinders in small spacing ratio is not bistable but monostable, which means that the biased flow phenomenon once formed, it remained in this pattern. Furthermore, Reynolds number has significant impact on the spacing ratio where the biased pattern disappears, it disappears at $L/D \geq 2$ for Re = 100 but $L/D \geq 3$ for Re = 200. The biased phenomena results in a large difference in the mean and r-m-s drag coefficients for the side by side downstream cylinders. (2) An interaction force exists in the gap of the side by side cylinders that may be an attractive or a repulsive force dominated by the pressure distribution in the gap of the cylinders. (3) At small spacing ratio, the fluctuating forces on downstream cylinder(s) are very small and far less than that on a single cylinder due to the proximity effect. But at large spacing ratio, they are much larger than that on a single cylinder due to the fluctuation of the shear layer from the upstream cylinder(s). Furthermore, the spacing ratio where the effect of the fluctuation reaches its maximum is related to the Reynolds number, $L/D = 4$ at Re = 200 but $L/D = 5$ at Re = 100.

1. Introduction

Cross-flow around a group of different arrangements of cylinders is involved in the port, shipping, construction, transportation, energy and other engineering fields. Offshore platforms pillar groups, heat exchanger tubes and chemical reaction towers are typical cross-flow around a group of cylinders. The flow-induced vibration may not only reduce the life of the equipment, but even lead to serious accidents as well. Therefore, in order to improve the life of the equipment and prevent such accidents from happening, it is very important to have a further understanding of the mechanism of flow-induced-vibration and the associated fluid-structure interaction. In the past three decades, relevant research mostly focused on flow around one or two cylinders but rarely on flow around more than two cylinders. In this paper, an equilateral-triangular arrangement of three cylinders with equal diameter is investigated for it is one of the most fundamental elements in any tube array and offshore structures. It is a well-known fact that the flow-induced vibration of cylinders has a close relationship with fluctuating forces affected by the vortex shedding and the characteristics of the flow patterns around the cylinders. Therefore, research on the forces of every cylinder in the equilateral-triangle arrangement of three cylinders and flow characteristics around the cylinders could enhance the understanding on the relationship between fluctuating forces and vortex shedding behavior around the cylinders.

In order to enhance the understanding of the flow pattern around three cylinders, classic flow properties of two cylinders should be mentioned: Zdravkovich [1] and Igarash [2] experimentally studied the flow pattern of two tandem cylinders with

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http://dx.doi.org/10.1016/j.compfluid.2016.02.013

different spacing ratios, they found that the spacing ratios has a significant impact on the flow pattern characteristics of the cylinders. Using a finite volume method, Chatterjee and Mondal [3–5] investigated the coupled fluid flow and heat transfer of two tandem equal isothermal square cylinders for Richardson number range of 0 to 2 and the Prandtl number is chosen constant as 0.71 at $50 \leq Re \leq 150$. The effect of superimposed thermal buoyancy on flow and isotherm patterns are presented and discussed in their studies. They also calculated and discussed the global flow and heat transfer quantities for various Reynolds numbers and spacing ratios. For side by side arrangement, when the spacing ratio ranges from 1.2 to 2.0, a bistable feature of the flow behind the cylinders is observed under some conditions, which is an important flow characteristics often resulting in a large difference in force and Strouhal number (St) of the two cylinders [6]. When the spacing ratio ranges from 2 to 6, the vortex shedding in anti-phase or in-phase occurs at the side by side cylinders. When the spacing is greater than 6, the flow around two cylinders approaches closely to that for a single cylinder. For the flow around three cylinders in an equilateral-triangular arrangement, it is extremely interesting to know whether the flow characteristics of two circular cylinders still exist or some new features may happen due to the effect of the third one.

Some experimental studies on the flow around three cylinders have been carried out. Using flow visualization, Gu and Sun [7] investigated the effects of spacing ratio on the flow characteristics on three cylinders in an equilateral triangle arrangement at $Re = 1.4 \times 10^4$. Tatsuno et al. [8] studied the effects of the flow interference between three cylinders in an equilateral arrangement with different incidence angles and spacing ratios at $Re = 6.2 \times 10^4$. Lam and Cheung [9] investigated vortex shedding and flow interference of three cylinders in different equilateral arrangements at $Re = 2.1 \times 10^4$ and $Re = 3.5 \times 10^4$. They found that the effect of the third cylinder was so significant that some well-known flow patterns in two cylinders arrays have been changed. Pouryoussefi et al. [10] measured the pressure distributions on the surface of three cylinders in an equilateral triangle arrangement at $Re = 6.08 \times 10^4$ and 1.26×10^4.

Those experimental studies of three cylinders in an equilateral arrangement were almost focused on fairly high Re, little is known of the phenomena at low Re. This is mainly because the measurement of experiments at low Re is difficult and inaccurate. Therefore, the numerical simulation has become a powerful tool for solving the complex cross-flow around multi-cylinders at low Re. Chatterjee et al. [11] studied the flow around a row of five square cylinders placed in a side-by-side arrangement at $Re = 150$ with four with four spacing ratios ranging from 1.2 to 4 by a finite volume method. Special attention is paid on the effect of the spacing between the five cylinders on the wake structure and vortex shedding mechanism. However, numerical studies of three cylinders in equilateral arrangement are still scarce relatively. Using a finite volume method, Chatterjee and Biswas [12] discussed the effect of transverse spacing on flow characteristics and global fluid dynamic parameters at $Re = 100$ with four spacing ratios ranging from 1 to 5; Chatterjee and Das [13] studied the effects of cross buoyancy and Prandtl number on the flow and heat transfer characteristics around three equal isothermal square cylinders arranged in a staggered configuration within an unconfined medium at $1 \leq Re \leq 30$ with a finite volume method. Bao et al. [14] used a split finite element method to simulate the flow around three equilateral arrangement at $Re = 100$ with six spacing ratios ranging from 0.5 to 4. Based on those studies, it is possible to understand the flow patterns and force characteristics on three cylinders at low Re, however, inadequate understanding of the flow physics still exists: (1) The spacing ratio in those studies ranging from 0.5 to 5, was not wide enough to cover large spacing ratio 6 which are widely adopted by experimental studies [9,10]. (2) The Reynolds number in those studies were limited to $Re \leq 100$, not cover $Re = 200$. It is well-known that 3-D instabilities appear in the wake of a circular cylinder at $Re \sim 190$ in some cases and some researchers have observed this phenomenon. If Re is much greater than 190, it is not suitable to use a 2-D model without proper correction. However, Lam [15] simulated the 2-D flow among four-cylinders at $Re = 200$ with different space ratio, and then by comparing the results with that of 3-D flow, he discovered that only at relatively small H/D (H/D = 16) will the obvious 3-D effect appears at the bottom of the cylinders, since the cylinders end walls might have some strong influence on flow pattern transition. Except for such cases, the results of the 2-D flow pattern are consistent with that of the 3-D flow pattern. This implies that the 3-D instabilities has little impact on the flow pattern of cylinders with the cylinders length being infinite at $Re = 200$. Thus, when the cylinders length are infinite, 2-D simulation is able to obtain acceptable results of the flow characteristics and force coefficients of the cylinders at $Re = 200$.

In order to provide a useful database of the force characteristics and a comprehensive understanding of the interaction of multi-cylinder arrays in a cross-flow at low Re, in this study, a two-dimensional finite volume method with unstructured mesh is used to perform a numerical simulation of cross-flow around three equal diameter cylinders arranged in an equilateral-triangular manner interact for two different angles, $\alpha = 0^\circ$ and $\alpha = 180^\circ$. Special attention is paid on the variation of flow field characteristics among the cylinders, force characteristics and Strouhal number of each cylinder with nine spacing ratios ranging from 1.5 to 7 at $Re = 100$ and 200, respectively.

2. Governing equations and numerical simulation

2.1. Basic governing equation and parameter settings

For a 2-D viscous incompressible unsteady flow, the governing equations can be written as follows:

Continuity equation:

$$\frac{\partial v_i}{\partial x_i} = 0 \tag{1}$$

Momentum equation:

$$\rho \frac{\partial v_i}{\partial t} + \rho v_j \frac{\partial v_i}{\partial x_j} + \frac{\partial P}{\partial x_i} - \mu \nabla^2 v_i = 0 \tag{2}$$

where v_i, ρ, t, p and μ represent the velocity component, the fluid density, the time, the pressure and the fluid dynamic viscosity, respectively. Einstein's summation convention is used in the subscript.

In this paper, a finite volume method based on the control volume technique is used to solve the general integral conservation form of the Navier–Stokes equation. The well-known SIMPLE technique is used to couple between the pressure and the velocity fields. The convective terms in the conservation equations is discretized with a second-order accurate upwind scheme. The diffusion term is discretized using a second-order central-difference technique, which is sufficient for the range of Re 100–200 investigated in this study. The reliability of those methods has been verified by Lam et al. [15].

These dimensionless parameters: the drag coefficient, the lift coefficient, the spacing ratio and the Strouhal number are defined as follows:

$$C_D = 2 \times \frac{F_d}{\rho u_\infty^2 D} \tag{3}$$

$$C_L = 2 \times \frac{F_l}{\rho u_\infty^2 D} \tag{4}$$