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A cell-centered multipoint flux approximation method with a diamond stencil coupled with a higher order finite volume method for the simulation of oil–water displacements in heterogeneous and anisotropic petroleum reservoirs

F.R.L. Contreras[a], P.R.M. Lyra[a], M.R.A. Souza[b], D.K.E. Carvalho[a,*]

[a] Department of Mechanical Engineering, UFPE, Av. Acadêmico Hélio Ramos s/n, CEP, 50670-901 Recife, PE, Brazil
[b] Department of Renewable Energy Engineering, UFPB, BR 230, s/n, CEP, 58059-900 João Pessoa, PB, Brazil

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ABSTRACT

The simulation of fluid flow and transport in heterogeneous and anisotropic oil and gas reservoirs poses a great challenge from the numerical point of view, due to the modeling of complex depositional environments, including inclined laminated layers, channels, fractures and faults and the modeling of deviated wells, making it difficult to build and handle the Reservoir Characterization Process (RCM), particularly by using structured meshes (cartesian or corner point), which is the current pattern in petroleum reservoir simulators. Under certain hypotheses, the mathematical model that describes the fluid flow in petroleum reservoirs includes an elliptic equation with heterogeneous, possibly discontinuous, coefficients for the pressure field and a non-linear hyperbolic equation for the saturation field. In the present paper, these equations are solved via an Implicit Pressure Explicit Saturation (IMPES) procedure. To solve these equations, we use a full cell-centered finite volume formulation. The pressure equation is discretized by a non-orthodox Multipoint Flux Approximation Method with a Diamond type stencil (MPFA-D) and used for the first time for the solution of two-phase flow problems in heterogeneous porous media. It is very robust and capable of reproducing piecewise linear solutions exactly by means of a linear preserving interpolation with explicit weights that avoids the solution of locally defined systems of equations. For the solution of the saturation equation, we use a Monotone Upstream Centered Scheme for Conservation Laws (MUSCL) method based on a gradient reconstruction obtained by a least square technique in which monotonicity is reinforced by an appropriate slope limiter. The method can be used with general polygonal meshes, even though we restrict ourselves to conforming triangular and quadrilateral grids. In order to validate and show the robustness of our formulation, we solve some problems including heterogeneous and anisotropic reservoirs and displacements with high mobility ratios. Our results compare well with others found in literature.

1. Introduction

The numerical simulation and consequent prediction of fluid flow patterns and transport in highly heterogeneous and anisotropic oil and gas reservoirs pose a great challenge for numerical algorithms, due to the modeling of complex depositional environments, including inclined laminated layers, channels, fractures and faults and the modeling of deviated wells, making it difficult to build and handle the whole Reservoir Characterization Process (RCM) [1], particularly by using structured meshes (Cartesian or corner point), which is the current standard in petroleum industry [2–9]. In complex petroleum reservoirs, heterogeneities range from pore size to reservoir scale [1,10] and rock properties, such as porosity and permeability can vary significantly throughout the reservoir. Under certain simplifying assumptions, the mathematical model that describes the fluid flow in petroleum reservoirs comprises an elliptic equation with a non-homogeneous and eventually discontinuous diffusion coefficient (i.e., permeability) for the pressure field and a non-linear hyperbolic equation for the saturation field which are weakly or strongly coupled by the velocity field depending on the solution procedure utilized.

Commonly, in petroleum reservoir simulators, the elliptic term (diffusive flux) associated to the pressure discretization is approximated by a simple Two Point Flux Approximation (TPFA) method

* Corresponding author. Tel.: +55 81 2126 8230; fax: +55 81 2126 8231.
E-mail addresses: ferlicapac@gmail.com (F.R.L. Contreras), dkarlo@oul.com.br, dkarlo@pq.cnpq.br (D.K.E. Carvalho).

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and the hyperbolic term (advective flux) associated to the saturation discretization is approximated via a simple First Order Upwind (FOU) method [7,8,11–13]. Even though this combined strategy is simple to implement and highly computationally efficient, it has certain drawbacks related to the inability of the TPFA method to properly discretize complex geometries that emerge from the geological modeling of the faulted and inclined structure of the reservoir and the anisotropic numerical diffusion introduced by the FOU method, which, not only, produces excessive spreading of the saturation fronts but is also prone to the so called Grid Orientation Effect (GOE) which represents a strong dependence of the numerical solution on the orientation of grid lines [12,14–19]. In order to face these problems, many strategies involving different combinations of numerical methods were presented in literature. Among the mass conservative schemes, we can quote the Mixed Finite Element (MFE) method, the Discontinuous Galerkin (DG) method [11,20,21] and the Multipoint Flux Approximation (MPFA) schemes [2–4,7]. Usually, in the MFE, pressure and velocities are approximated simultaneously with the same order of accuracy and the saturation equation is solved by means of some shock capturing method [8,22]. On the other hand, DG methods have been used to solve both: the pressure equation, by means of symmetric or non-symmetric interior penalty methods; and the saturation equation, together with some form of higher order upwinding and limiting to ensure monotonicity [11,20,21]. Recently, classical MPFA methods coupled with higher order upwind or central schemes have been used to simulate two phase flows in porous media [18,23]. In [10] a node centered multipoint flux approximation method with full pressure support (MPFA-FPS) was used together with a higher order locally conservative and non-split convective scheme to simulate oil and water displacements in porous media. Due to the truly multidimensional and higher order character of these methods, they proved to be very robust for various test problems involving full tensor and heterogeneous permeability fields in structured and unstructured quadrilateral meshes.

In this context, in the present paper we devise a full cell-centered finite volume procedure to solve both, the pressure and the saturation equations, via an IMplicit Pressure-Explicit Saturation (IMPES) technique which was originally devised by Sheldon and Cardwell (1959) and Stone and Gardner (1961) and has being extensively used in practice for moderate complexity multiphase flow problems in petroleum reservoirs [5,8,14,18,24–28]. In this technique, a sequential time stepping procedure is used to split the computation of the pressure from the saturation fields. In the classical IMPES approach, starting from an initial saturation distribution, the pressure equation is solved implicitly and then, the total velocity is calculated explicitly from this pressure field. Following, the velocity field is used to compute the advective fluxes in the saturation equation, which is solved explicitly, and the process is repeated until the end of the simulation.

In our work, the pressure equation is discretized by a non-orthodox cell-centered Multipoint Flux Approximation Method with a Diamond type stencil (MPFA-D) which was originally devised by [29] to solve diffusion problems in heterogeneous and anisotropic media using polygonal meshes. In this scheme, the flux on each control volume face (edge in 2-D) is explicitly expressed by two cell-centered unknowns defined on the control volumes (CVs) sharing that edge and two auxiliary unknowns defined at the two edge endpoints. As the scheme is cell-centered, vertex variables are expressed as weighted linear combinations of the neighboring cell-centered unknowns in order to reduce the scheme to a completely cell-centered one. As pointed out by [29], there are different ways to compute the weights for the vertex unknowns, for instance, by using Taylor expansions [30], by a straightforward bilinear interpolation [31] or its modification by the finite point method [32]. In the method of [29] the weights are either discontinuity or mesh topology dependent as in some of the other previous methods and can be used even for full tensor problems. The derivation of the scheme and of these weights satisfy the linearity preserving criterion, which requires that a discretization scheme should be exact on piecewise linear solutions [5,6,29]. In the present paper, we adapt the method for the solution of two-phase flows in petroleum reservoirs by computing the total mobility term on the control surface as a function of the mobilities of the CVs that share at least a node with the surface. To solve the non-linear saturation equation we use a modified version of the second order upwind type, vertex centered finite volume method devised by [33] adapted to our cell-centered method. Second order accuracy is obtained via a least square gradient reconstruction and monotonicity is enforced by using slope limiters following a MUSCL strategy [34–38]. In the present paper, we restrict ourselves to triangular and quadrilateral meshes, but our method can be used with general polygonal meshes as the gradient reconstruction and the monotonicity constraints are independent of the shapes of the CVs. In order to validate our formulation, i.e., the use of the MPFA-D for the pressure field, combined with the high order MUSCL-type finite volume method (HOMFV) for the saturation one, we simulate oil–water displacements with moderate or high mobility ratios, in heterogeneous and anisotropic (full tensor) petroleum reservoirs using structured and unstructured triangular and quadrilateral meshes.

2. Mathematical formulation

In this section, we briefly describe the governing equations for the two-phase flow of oil and water in petroleum reservoirs. We assume, without loss of generality, that the fluid and rock are both incompressible, that the flow is isothermal and we neglect the capillary pressure and gravitational term. We will use a segregated formulation in which the basic equations are obtained from the proper combination of the conservation of mass and the Darcy's Law, which can be written for phases i =*o (oil), w (water)*, respectively, as:

$$\frac{\partial(\phi\rho_i S_i)}{\partial t} = -\nabla\cdot(\rho_i\vec{v}_i) + q_i \tag{1}$$

$$\vec{v}_i = -\lambda_i \underset{\sim}{K}\nabla p_i,\ i = o, w \tag{2}$$

In Eqs. (1) and (2), ϕ is the rock porosity, ρ_i and S_i, represent, respectively, the density and the saturation of phase i, i.e., the fraction of the pore volume occupied by phase i, $\vec{v}_i$, is the phase velocity, which is given by Darcy's law and q_i denotes source or sink terms (e.g., injection or production wells) and $\underset{\sim}{K}$ is the absolute rock permeability tensor that satisfies the ellipticity condition and the fluid mobility is given by $\lambda_i = k_{ri}/\mu_i$, where μ_i and $k_{ri}(S_i)$ represent the viscosity and the relative permeability of phase i, respectively. We also assume that the reservoir rock is fully saturated by oil and water. Due to this last assumption, we can write:

$$S_o + S_w = 1 \tag{3}$$

By using Eqs. (1)–(3) and after some algebraic manipulation [8,13,14,25,39], we can write the elliptic pressure equation, as:

$$\nabla\cdot\vec{v} = Q \quad \text{with} \quad \vec{v} = -\lambda\underset{\sim}{K}\nabla p \tag{4}$$

In which the total mobility is $\lambda = \lambda_w + \lambda_o$.

In Eq. (4) the total fluid velocity is denoted by $\vec{v} = \vec{v}_w + \vec{v}_o$. The total fluid injection or production specific rate is denoted by $Q = Q_w + Q_o$ with $Q_i = q_i/\rho_i$. Again, by using Eqs. (1)–(3) and after some algebraic manipulation [8,13,14,25,39], we can write the hyperbolic saturation equation, as:

$$\phi\frac{\partial S_w}{\partial t} = -\nabla\cdot\vec{F}(S_w) + Q_w \tag{5}$$