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A Kalman filter adapted to the estimation of mean gradients in the large-eddy simulation of unsteady turbulent flows

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ABSTRACT

A computationally-efficient method based on Kalman filtering is introduced to capture "on the fly" the low-frequency (or very large-scale) patterns of a turbulent flow in a large-eddy simulation (LES). This method may be viewed as an adaptive exponential smoothing in time with a varying cut-off frequency that adjusts itself automatically to the local rate of turbulence of the simulated flow. It formulates as a recursive algorithm, which requires only few arithmetic operations per time step and has very low memory usage. In practice, this smoothing algorithm is used in LES to evaluate the low-frequency component of the rate of strain, and implement a shear-improved variant of the Smagrosinky's subgrid-scale viscosity. Such approach is primarily devoted to the simulation of turbulent flows that develop large-scale unsteadiness associated with strong shear variations. As a severe test case, the flow past a circular cylinder at Reynolds number $\mathrm{Re}_D = 4.7 \times 10^4$ (in the subcritical turbulent regime) is examined in details. Aerodynamic and aeroacoustic features including spectral analysis of the velocity and the far-field pressure are found in good agreement with various experimental data. The Kalman filter suitably captures the pulsating behavior of the flow and provides meaningful information about the large-scale dynamics. Finally, the robustness of the method is assessed by varying the parameters entering in the calibration of the Kalman filter.

1. Motivations

The numerical simulation of turbulent flows in geometries of engineering interest can be accomplished with various levels of approximation, yielding a more or less detailed representation of the flow. The so-called direct simulation, in which the equations of motion are discretized and solved directly, is obviously the most straightforward approach. If the mesh is sufficiently fine to resolve even the smallest eddies, and if the numerical scheme limits dispersion and dissipation errors, this method yields an accurate time-dependent representation of the flow [1]. Unfortunately, its applicability is limited to simple geometries at relatively low Reynolds numbers. The reason is twofold. First, the drawback of using highly accurate schemes is unavoidably a lack of flexibility to handle complex geometries and general boundary conditions. Second, the resolution of turbulent fluid motions at high Reynolds numbers requires a prohibitive number of grid points, especially in near-wall regions where thin vortical structures develop [2]. Therefore, in practical situations, the direct approach is often abandoned in favor of approximate, but numerically tractable, computations.

In a large-eddy simulation, usually referred to as LES in the literature, the grid resolution is deliberately reduced so that only the large-scale motions of the fluid are captured numerically. This is physically justifiable since the large-sized eddies contain most of the kinetic energy of the flow, and their strengths make them the efficient carriers of mass, momentum, heat, etc. On the contrary, small-sized eddies are mainly responsible for dissipation and contribute little to transport and mixing. The large-scale dynamics is solution of the original flow equations, *e.g.* the Navier–Stokes equations, supplemented by an unknown term accounting for the stress exerted by the unresolved subgrid-scale motions on the simulated flow. A common thread is to assume that this stress is essentially responsible for a diffusive transport of fluid momentum at grid scale, which in turn calls for the modeling of a subgrid-scale viscosity [3]. This viscosity depends on space and time, and is related to the (subgrid-scale) turbulent dynamics.

In the context of engineering flows, which may experience strong unsteady events such as boundary-layer separation, vortex shedding or disturbances induced by a moving body, *e.g.* a turbine blade, the modeling of the subgrid-scale viscosity is recognized to be a difficult problem. Strong unsteadiness generally occurs at low frequencies in comparison with turbulent fluctuations in the bulk and is often associated with large amplitudes of the rate of strain (or shear). In this respect, a refinement of the Smagorinsky's model [4] has been

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proposed recently. Namely, the so-called shear-improved Smagorinsky's model (SISM) [5] accounts explicitly for the mean part (in the sense of statistical average) of the rate of strain to correct the Smagorinsky's viscosity. The resulting viscosity expresses as

$$\nu_{\text{sgs}}(\mathbf{x},t) = (C_s\Delta(\mathbf{x}))^2(|\overline{S}(\mathbf{x},t)| - \mathcal{S}(\mathbf{x},t)), \quad (1)$$

where $C_s = 0.18$ is the standard Smagorinsky constant [6], $\Delta(\mathbf{x})$ is the local grid spacing (at position $\mathbf{x}$) and $|\overline{S}(\mathbf{x},t)|$ is the norm of the resolved rate-of-strain tensor: $|\overline{S}| = \sqrt{2\sum_{ij}\overline{S}_{ij}\overline{S}_{ij}}$. In the notation, the overline recalls that the flow quantity is discretized at a grid resolution that may be coarse compared to the size of the smallest turbulent eddies. In Eq. (1), the correcting term to the Smagorinsky's viscosity is $\mathcal{S}(\mathbf{x},t) = |\widetilde{\overline{S}}x,t)|$, where the tilde refers typically to a low-pass filtering (as discussed below). Interestingly, the SISM does not call for any adjustable parameter besides $C_s = 0.18$, which is fixed for all flows. There is no need for an *ad-hoc* damping function nor any kind of dynamic adjustment in near-wall regions [7]. The simplicity and manageability of the original Smagorinsky's model are therefore preserved.

The theoretical basis of the SISM was put forward on account of numerical and experimental studies on shear effects in non-homogeneous turbulence [9].

In the context of subgrid-scale modeling, it shares obvious similarities with the model originally introduced by Schumann in 1975, which relies on a two-part eddy-viscosity accounting for the interplay between the nonlinear energy cascade present in isotropic turbulence and mean-shear effects associated with anisotropy [10]. However, the SISM clearly differs from Schumann's proposal. This later requires an empirical prescription for the "inhomogeneous eddy-viscosity", whereas the subgrid-scale viscosity is explicit in the SISM and arises naturally from the scale-by-scale energy budget established from the Navier–Stokes equations [5]. Another important point is that the SISM cannot be obtained by just simplifying Schumann's formulation. Let us note that variants of Schumann's model have also been proposed by Moin and Kim in 1982 [11] and followed by Horuiti in 1987 [12], and one can add the anisotropic version introduced by Sullivan *et al.* in 1994 [13].

Including anisotropy effects in the SGS modeling has been addressed in many different ways. The SISM relies on a decomposition of the resolved flow into a statistically-averaged part and a fluctuating part. An alternative decomposition into a large-scale and a small-scale component has been extensively explored. This refers for instance to the variational multi-scale (VMS) method, which originates with the works of Temam on multi-level methods [14] and has been developed by Hughes *et al.* [15] and many others thereafter (see [16] for a review). This decomposition arises from the motivation to build an eddy-viscosity on either the small-scale or the large-scale part of the velocity field, and make it act on the small-scale part of the resolved motions only. One can also mention the filtered structure-function model introduced by Ducros *et al.* [17] that consists of removing the large-scale fluctuations of the velocity field before computing its second-order structure function.

An important requirement of the SISM is to evaluate the mean component of the rate of strain (in the sense of ensemble average) as the simulation progresses. In practice, ensemble average may be approximated by space average over directions of homogeneity, whenever it is possible, *e.g.* in a plane-channel flow. When it is not, time average may be used instead if the flow is statistically time-invariant. However, many engineering flows do not allow such approximations and an alternative estimation must be found, which is the issue addressed in the present work. Our proposal is to assume that the mean flow may be approximated by the low-frequency component of the velocity field, including a possible (quasi-)deterministic unsteadiness, and that the turbulent component adds itself to this "unsteady" mean (see [8] for more details). An original method based on Kalman filtering in the time domain is investigated. Importantly, this filter corrects its cut-off frequency automatically according to the local turbulent rate of the flow. It is therefore well-adapted to strongly inhomogeneous and unsteady flows. This method is fully local in space and applies independently at each grid point. It is thus convenient to treat complex-geometry flows, possibly integrated on unstructured grids. The physical fundamentals of this method have already been presented in a companion paper [8], the focus is here on computational and validation aspects, including a spectral analysis.

2. Kalman filtering adapted to turbulent flows

2.1. Exponential smoothing as baseline method

A simple way to extract the low-frequency component of a digital signal is to apply a weighted moving average (in time) to this signal. In the context of computational fluid dynamics, this moving average should be applied at each grid point and every time step, making the cost of this operation highly selective. The simplest solution is certainly to consider an exponentially-weighted moving average, or exponential smoothing [18,19]. See [20–22] for existing applications of exponential smoothing in the context of LES. The main advantage of the exponential smoothing is that it can be formulated in a very convenient recursive manner:

$$\widetilde{\overline{\mathbf{u}}}^{(n+1)} = (1-\alpha)\cdot\widetilde{\overline{\mathbf{u}}}^{(n)} + \alpha\cdot\overline{\mathbf{u}}^{(n+1)}, \quad (2)$$

where $\widetilde{\overline{\mathbf{u}}}^{(n)}$ denotes the smoothed velocity (at time n) whereas $\bar{\mathbf{u}}^{(n)}$ is the instantaneous velocity. The smoothing factor $0 < \alpha < 1$ controls the weights of the past observations in the average (a higher α discounts older observations faster). The exponential smoothing is formally equivalent to a first-order low-pass filter with a cut-off frequency f_c related to the smoothing factor by

$$\alpha \simeq \frac{2\pi f_c\Delta t}{\sqrt{3}} \approx 3.628 f_c\cdot\Delta t, \quad (3)$$

where Δt is the time step of the velocity signal (see [8] for a proof).

In the exponential smoothing, the key point is to update at each time step the smoothed quantity (here the velocity) by taking into account the new data point. It is computationally efficient since it requires only the storage of the (previous) smoothed quantity. Also, the initialization of the algorithm is very simple: $\widetilde{\overline{\mathbf{u}}}^{(0)} = \overline{\mathbf{u}}^{(0)}$. In the context of complex turbulent flows, an obvious limitation of this method is to select a unique physically-relevant cut-off frequency for the whole flow. In practice, the smoothing factor is expected to vary in space and time according to the large-scale inhomogeneity and unsteadiness of the flow. In the following, it is shown that this limitation can be alleviated by considering an adaptive exponential smoothing, in which the smoothing factor $\alpha(\mathbf{x},t)$ adjusts itself automatically according to the local turbulent rate of the velocity field. This procedure is made possible by means of an *adaptive* Kalman filter. Integrating Kalman filtering in the SISM therefore allows us to extend the scope of this sudgrid-scale model to the LES of inhomogeneous and unsteady turbulent flows.

2.2. Adaptive exponential smoothing based on Kalman filtering

A Kalman filter estimates the state of a dynamical system, here the low-frequency component of the velocity field, from a series of observations. Kalman filtering is a major topic in control theory in engineering science and is known to be rather efficient [23,24]. As for the exponential smoothing, an important feature of a Kalman filter is its formulation as a recursive estimator. The updated state is computed from the previous state and the current observation only. In our case, the update is made according to Eq. (2) but with a smoothing factor (noted K) that is now inferred dynamically from the local fluctuation of the signal. This inference is performed on the basis