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Time-harmonic Navier–Stokes computations of forced shock-wave oscillations in a transonic nozzle

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ABSTRACT

This study focuses on the development of frequency-based Reynolds-Averaged Navier–Stokes methods in the presence of harmonic excitations. Two different methodologies are proposed to alleviate the problem of high computational costs of conventional time-domain time-nonlinear approaches due to the capture of the long transients. A time-linearized approach is adopted using either the simple frozen-turbulence-scales assumption or the full linearization of the turbulence model. In order to account for nonlinear coupling between harmonics, a flexible time-domain Fourier-based solver is derived from a Reynolds-Averaged Navier–Stokes solver based on a local dual time stepping technique. Various flow regimes, involving forced shock-wave oscillations due to an elliptical cam placed at the nozzle exit and forced vibrations of test objects, are investigated to assess the robustness and the computational efficiency of the two frequency-based approaches in the presence of recirculating flows.

1. Introduction

The prediction of the flutter onset speed is of crucial importance in the field of turbomachinery aeroelasticity. The corresponding loss of dynamic stability, which results in unbounded vibrations of the structure, may lead to dramatic mechanical failures. Transonic stall flutter can be accurately computed with conventional time-nonlinear Reynolds-Averaged Navier–Stokes (RANS) solvers combined with advanced turbulence closures. However, such time-domain formulations exhibit prohibitive computational costs due to the long transients required to obtain a time-periodic solution. During the last decades, many authors focused on frequency-domain based methods as an alternative to the solution of the unsteady Navier–Stokes equations in time-domain, such as the Time-Linearized RANS (LRANS) and the Time Harmonic Balance RANS (THB-RANS) approaches. The former assumes that the unsteady flow can be considered as a small time-harmonic perturbation superimposed on an underlying steady flow while the latter is based on the Fourier decomposition of the flow variables and residuals.

The two dimensional LRANS equations were successfully solved by Clark [27,28,62] for aerodynamic damping and stall flutter computations. Then, numerous three-dimensional LRANS solvers were developed for turbomachinery flutter [15,17,19,20,60,66,75,80] and aircraft aeroelasticity [30,68,73]. To this end, different approaches were proposed to deal with the unsteady perturbations in the turbulent stresses, such as the frozen eddy-viscosity assumption [25], the frozen-turbulence-scales approximation [21], the linearized form of the one equation Spalart–Allmaras model [15,28,80] or the full linearization of the $k-\omega_{\mathrm{T}}$ model [60]. However, these approaches may suffer from a lack of robustness due to the unbounded amplification of the perturbation variables in pseudo-time [1,15,27,30,31,60]. The relationship between the numerical instabilities and small perturbations of the base flow field was highlighted by Campobasso and Giles [15] for transonic viscous flows in turbomachines. In particular, complex conjugate pairs of outliers, which are responsible for the exponential growth in pseudo-time, were related to flow phenomena involving separation bubbles [15]. Stabilization techniques of the linear code were successfully developed by implementing a GMRES algorithm [15] or using a recursive projection method [16,17]. An alternative to the use of stabilization approaches is to employ a direct solver for the solution of the linearized Euler or Navier–Stokes equations [1,22,77].

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On the other hand, He [55,56] and later He and Ning [59,74] proposed to account for nonlinear coupling effects between the time-averaged flow and the first harmonic unsteady perturbation for the computation of the flow response around oscillating blades. This approach was extended to higher harmonics by Hall et al. [53] to compute time-periodic flows in cascades by solving the coupled set of equations related to each harmonic of the Fourier decomposition in the time-domain. The THB-RANS method was found to be very efficient in the computation of single frequency dominated flows [30,47,49,50,82,83,85] and unsteady flows in the presence of multiple frequencies [32,34,58,84,94]. Recently, a multi-frequency THB-RANS approach with non-uniform time sampling was employed by Sicot et al. [52,84] for robust and accurate aeroelastic computations of a counter-rotating fan. A thorough convergence analysis of Fourier-based time methods for turbomachinery wake passing problems was presented in [46]. Extensive review of the frequency-domain approach are given in [57,71]. One of major advantages of the harmonic balance method over its time-linearized counterpart, is that it can be developed within existing implicit steady flow solver codes with minimum coding efforts [6,85,91,97]. Therefore, such an approach was recently deployed for a wide range of applications in aeronautical and aerospace engineering, like for instance, turbomachinery blade rows interactions [35–37,63,83] and flutter [10,33,89], rotorcraft applications [26,64,69,98], wind turbine [14,61], and aircraft aeroelasticity [90]. A detailed comparative study between LRANS and THB-RANS was performed by Dufour et al. [30] for a Naca64A006 airfoil with a 75% chord oscillating flap. Quite an accurate description of the unsteady field were obtained using the THB-RANS approach with two harmonics for the transonic recirculating case with a speed-up about three compare to the URANS solution. On the contrary the LRANS with pseudo-time failed to converge. The linear and the nonlinear frequency approaches were compared in the context of Euler flows by Da Ronch et al. [79] and by Woodgate and Barakos [98] for Navier–Stokes computations of rotor flows.

In this work, we perform a comparative study between LRANS and THB-RANS approaches, along the lines of Dufour et al. [30], for the computation of forced oscillations in a transonic nozzle. New advances are proposed to circumvent the problem of lack of robustness and accuracy in the presence of separated flows. First, we solve the linear system resulting from the discretization of the LRANS equations by means of Krylov's algorithms without pseudo-time-marching in order to avoid convergence problems. The unsteady turbulent stresses are computed by means of a frozen turbulence scale approach or by using a derived turbulence model. Second, an efficient time-domain Fourier-based solver with Reynolds-stress closure [40,41,44] is developed from a steady Reynolds-Averaged Navier–Stokes solver [45,93]. In order to validate our numerical results, we consider the experimental nozzle facility at the Chair of Heat and Power Technology of KTH [3–5,11,13,95], which is based on a simplified aeroelastic test case bringing into focus the area of interaction between an oscillating shock wave and a turbulent boundary layer. The unsteady flow response is computed for back-pressure fluctuations due to an elliptical cam placed downstream of the test section and forced vibrations for excitation frequencies ranging from 100 Hz up to 500 Hz.

The layout of the paper is as follows: Section 2 presents the computational framework and the development of the frequency-domain based RANS numerical solvers. The different formulations of the unsteady flow problem are applied to time-periodic separated flows in Section 3. Finally, Section 4 draws the concluding remarks.

2. Governing equations and computational methods

2.1. Computational framework

In this work, the unsteady flow due to deterministic forcing is modeled using the compressible Favre–Reynolds averaged Navier–Stokes equations

$$\frac{\partial \underline{w}}{\partial t} + \mathrm{div}\left[\vec{\underline{F}}^C(\underline{w}) - \vec{\underline{F}}^V(\underline{w})\right] + \underline{S}(\underline{w}) = 0 \quad (1)$$

where $\underline{w}$ denotes the vector of the conservative variables, $\vec{\underline{F}}^C$ and $\vec{\underline{F}}^V$ are the convective and viscous fluxes respectively and $\underline{S}$ represents the vector of source terms due to turbulence closure. The computational framework which will be used to establish both LRANS and THB-RANS formulations is based on the structured multi block cell-vertex finite volume solver developed by Vallet [93] with near-wall Reynolds-stress closure [40,41,44,45]. The governing equations are solved using a van Leer flux-vector-splitting scheme with third order MUSCL interpolations and van Albada limiters [42,43,45]. A local dual time stepping (LTDS) procedure is employed to improve the convergence properties of the steady flow solution in the presence of separated flows, resulting in dramatic reduction of limit-cycle-oscillations due to the use of approximate Jacobian matrices and ADI factorization [23]. An implicit $O(\Delta t^2)$ dual time stepping technique is used to solve the unsteady RANS equations in time-domain [24]. This time-nonlinear approach, namely referred as TNL-RANS hereafter, will be used for validation purpose of the frequency-domain solvers developed in Sections 2.2 and 2.3.

2.2. Time-linearized Navier–Stokes equations

The first step in the derivation of the time-linearized time-harmonic equations is to consider that the unsteady flow can be modeled as a steady flow ${}^0\underline{w}(\vec{x})$ plus a small harmonic perturbation ${}^1\underline{w}(\vec{x},t)$. Therefore the conservative variables may be expressed as [54]

$$\underline{w}(\vec{x},t) = {}^0\underline{w}(\vec{x}) + {}^1\underline{w}(\vec{x},t) = {}^0\underline{w}(\vec{x}) + \Re\left[{}^1\hat{\underline{w}}(\vec{x})e^{i\omega t}\right] \quad (2)$$

where ${}^1\hat{\underline{w}}(\vec{x})$ represents the harmonics of the unknowns and $\omega = 2\pi f$ is the pulsation related to the perturbation frequency f. Next, the decomposition given by (2) is introduced into the governing equations (Eq. (1)). Collecting the zero and first order terms and neglecting the high order terms, gives the following formulation of the Navier–Stokes equations in the frequency-domain [21]:

$$\mathrm{div}\left[{}^0\vec{\underline{F}}^C(\underline{w}) - {}^0\vec{\underline{F}}^V(\underline{w})\right] + {}^0\underline{S}(\underline{w}) = 0 \quad (3)$$

$$i\omega\,{}^1\hat{\underline{w}} + \mathrm{div}\left[{}^1\hat{\vec{\underline{F}}}^C({}^0\underline{w},{}^1\hat{\underline{w}}) - {}^1\hat{\vec{\underline{F}}}^V({}^0\underline{w},{}^1\hat{\underline{w}})\right] + {}^1\hat{\underline{S}}({}^0\underline{w},{}^1\hat{\underline{w}}) = 0 \quad (4)$$

The underlying steady flow recovered by (3) is solved by the numerical method depicted in Section 2.1. The linearization strategy used to derive the numerical scheme associated to (4) consists in linearizing the discretized nonlinear equations [21]. To this end, we compute the Jacobian matrices of the convective and viscous numerical fluxes as thoroughly described in [18,93].

The LRANS solver described in Section 2.2 employs a frozen turbulence scales approximation. The linearization of turbulent stresses is addressed by assuming a Boussinesq hypothesis for the unsteady perturbation of the turbulent stresses [21]. To this end, the unsteady fluctuations of the turbulence-Reynolds-number are neglected. As a consequence, the perturbations in the eddy viscosity is directly related to the perturbation in the molecular viscosity.