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Drag, lift and torque coefficients for ellipsoidal particles: From low to moderate particle Reynolds numbers

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An accurate prediction of the translational and rotational motion of ellipsoidal particles can be only given if a complete set of correlations of the drag, lift and pitching torque coefficients is known. The present study is thus devoted to the assessment of the available correlations in the literature through a comparison with numerical results of the forces acting on a particle given by a full body-fitted direct numerical simulation (DNS) in the case of a uniform flow, for three different ellipsoidal particles, and for Reynolds number ranging from 0.1 to 290. The comparison between the computed force (or hydrodynamic coefficients) and the literature correlations shows clearly that certain precautions must be taken. The mean deviation of the most accurate correlation considered in the present study from our full DNS results can be of the order of 20% for the drag coefficient. For the lift and pitching torque coefficients, the deviations can increase up to roughly 25%. This comparison shows that further works are definitively necessary to develop a complete set of correlations for ellipsoidal particles outside Stokes regime.

1. Introduction

The assumption that particles are spherical has been extensively used in the modeling of particle transport in fluid flows for the past 20 years (Marchioli et al. [1], Arcen et al. [2]). However, in natural processes as in a variety of engineering applications such as dispersion of pollutant, pulverized coal combustion, pneumatic transport, or in all aspects of the pulp and paper industry, many different shapes are possible. Owing to the significant effect of the particle shapes on their motion and their practical importance in hydrodynamic applications, the non-sphericity is more and more considered in the modeling and simulations of particles transport in fluid flows (Mandø and Rosendahl [3], Lain and Sommerfeld [4], Njobuenwu and Fairweather [5], van Wachem et al. [6]). Unfortunately, it is not possible to consider each shape in the implementation of numerical models, because of the nonexistence of a single approach describing accurately the sizes and shapes of non-spherical particles. Therefore, irregular particle shapes are commonly idealized to some regular shapes such as spheroids, cuboids or cylinders (see Fig. 1). In addition, several methods of size measurement, shape categorization, and shape parameters used to describe regular and irregular non-spherical particles were previously proposed and summarized in Clift et al. [7], Allen [8], Mandø [9], and Li et al. [10].

Even if the particle shapes are idealized, a complete description of the behavior of these particles requires the modeling of their translational and rotational motion taking its orientation into account, in contrast to the spherical particle case.

Several studies have been performed over the past 20 years, but they are mainly devoted to the drag force acting on non-spherical particles. Chhabra et al. [13] were the first to compile and classify the most used correlations. According to their study they showed that the correlation derived by Ganser [14] is the most accurate. They distinguished two types of approach that were used in later studies. The first one is to define a single correlation for all shapes and orientations (Tran-Cong et al. [15], Yow et al. [16], Holzer and Sommerfeld [17]). Indeed, Tran-Cong et al. [15] proposed a correlation from their experimental data obtained from agglomerates of ordered packed spheres. Yow et al. [16] used data available in the literature to extract a new correlation which unfortunately, does not take the orientation into account. This parameter was introduced in the correlation of Holzer and Sommerfeld [17], through two shape parameters which are the sphericity (ϕ) and the crosswise sphericity. The sphericity is defined by Wadell [18] as the ratio between the surface area of the volume equivalent sphere and the surface area of the considered particle (S). The crosswise sphericity, $\phi_{\perp}$, is defined by Holzer and Sommerfeld

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[17] as the ratio between the cross-sectional area of the volume equivalent sphere and the projected cross-sectional area of the considered particle perpendicular to the flow ($S_\perp$). The second approach consists in obtaining a drag coefficient expression for a fixed shape and any orientations. Rosendahl [19] suggested to determine the drag coefficient at two incidence angles (α), *i.e.* 0° and 90° ($C_{D,\alpha=0^\circ}$, $C_{D,\alpha=90^\circ}$), from the best experimental results or from existing correlations. Then, a function linking these two limiting cases, with an inflection point at 45°, is used to represent the whole range of incidence angles for a non-spherical particle. Recently, Zastawny et al. [20] proposed new correlations to predict the drag, lift, and torque (pitching and rotational) coefficients for non-spherical particles from data given by a direct numerical simulation carried out with an immersed boundary method. Their correlation for the drag force is derived following a functional form similar to that proposed by Rosendahl [19], nonetheless $C_{D,\alpha=0^\circ}$ and $C_{D,\alpha=90^\circ}$ are given from formulas which are function of the particle Reynolds number and particle-specific fit parameters. In their study, Zastawny et al. [20] compared their drag coefficient for ellipsoidal particles with the theoretical expression provided by Happel and Brenner [21] at low Reynolds numbers. At higher Reynolds numbers, the comparison is conducted with the formula proposed by Rosendahl [19], the drag coefficient at 0° and 90° being given by the correlation proposed by Holzer and Sommerfeld [17]. At low and higher Reynolds numbers, some significant differences were noted. Currently, given this uncertainty on the accuracy of the exiting drag coefficient correlations for ellipsoidal particles, it is difficult to make a choice about the drag correlation to use. This choice is all the more difficult for the lift or torque coefficients since there are few studies devoted to these coefficients in the literature. In some studies, a proportionality of the lift coefficient with the drag one is assumed (Yin et al. [22], Hoerner [23]). Nevertheless, Zastawny et al. [20] showed the weakness of this assumption through their DNS results and provided a new correlation valid for ellipsoidal particles and for two aspects ratios. For instance, they proposed the following relation for the lift coefficient (Table 1 includes all the correlations proposed by Zastawny et al. [20]):

$$C_L = \left(\frac{b_1}{Re_p^{b_2}} - \frac{b_3}{Re_p^{b_4}}\right)\sin(\alpha)^{b_5+b_6 Re_p^{b_7}}\cos(\alpha)^{b_8+b_9 Re_p^{b_{10}}}, \quad (1)$$

where the coefficients, b_i, are specific to the aspect ratio. At the present time, there is no correlation of the lift coefficient which can be applied for any particle aspect ratio. Concerning the pitching torque, it is generally accepted that it is intimately linked to the drag and lift forces which act at the pressure center, its location being different from the gravity center, this induces the pitching torque. The only formula for the pitching torque coefficient was given by Zastawny et al. [20] for ellipsoidal particles and for specific values of the aspect ratio.

Up to now, there is no set of correlations of the drag, lift and pitching torque coefficients for ellipsoidal particles outside the Stokes regime which takes into account the incidence angle, aspect ratio, and Reynolds number. Therefore, when the motion of ellipsoidal particles has to be predicted, some choices have to be made among available correlations even if they were derived under strong assumptions. For instance, several authors studied the motion of inertial ellipsoidal particles using the theoretical expression for the force and torque derived under the assumption of low Reynolds number (Zhang et al. [24], Mortensen [25], Marchioli et al. [26], Tian and Ahmadi [27], Marchioli and Soldati [28], Zhao et al. [29]). The torque was obtained from the expression derived by Jeffery [30] while the force was computed from the expression given in Happel and Brenner [21]. However, when the particle inertia is large, the low Reynolds number assumption is not valid anymore. Due to the lack of correlations outside the Stokes regime, researchers are therefore generally forced to study the non-spherical particle motion under the creeping flow assumption. An accurate prediction of the complete motion of non-spherical particles in a turbulent flow by a Lagrangian approach and outside the Stokes regime would be possible if an accurate set of hydrodynamic coefficient correlations was available. The aim of the present work is thus twofold. Firstly, to guide the reader through the choice of correlations of the drag, lift and torque coefficients outside the Stokes regime. Secondly, to provide data on the drag, lift, and pitching torque coefficients for different ellipsoidal particles by a full direct numerical simulation using a body-fitted approach. It is hoped that this database will help to develop models of these coefficients.

The document is organized in three sections. The first one is devoted to the description of the model of the motion of non-spherical particles and to the main available correlations. In the second section, the numerical simulation technique and some the validation cases are presented. Then, the results obtained for the non-dimensional hydrodynamical parameters (drag, lift, and pitching torque coefficients) are given, for three different particle shapes, and discussed in the third section before the conclusion.

Table 1
Drag, lift, and torque correlations.

Authors	Correlations
Holzer and Sommerfeld [17]	$C_D = \frac{8}{Re_p}\frac{1}{\sqrt{\phi_\perp}} + \frac{16}{Re_p}\frac{1}{\sqrt{\phi}} + \frac{3}{\sqrt{Re_p}}\frac{1}{\phi^{3/4}} + 0.42\times 10^{0.4(-\log\phi)^{0.2}}\frac{1}{\phi_\perp}$ ϕ is the sphericity and $\phi_\perp$ the crosswise sphericity
Rosendahl [19]	$\begin{cases} C_D = C_{D,\alpha=90^\circ} + (C_{D,\alpha=90^\circ} - C_{D,\alpha=0^\circ})\sin^3\alpha \\ \text{Where :} \\ C_{D,\alpha=0^\circ} = \frac{8}{Re_p}\frac{1}{\sqrt{\phi_\perp}} + \frac{16}{Re_p}\frac{1}{\sqrt{\phi}} + \frac{3}{\sqrt{Re_p}}\frac{1}{\phi^{3/4}} + 0.42\times 10^{0.4(-\log\phi)^{0.2}}\frac{1}{\phi_\perp} \\ C_{D,\alpha=90^\circ} = \frac{8}{Re_p}\frac{1}{\sqrt{\phi_\perp}} + \frac{16}{Re_p}\frac{1}{\sqrt{\phi}} + \frac{3}{\sqrt{Re_p}}\frac{1}{\phi^{3/4}} + 0.42\times 10^{0.4(-\log\phi)^{0.2}}\frac{1}{\phi_\perp} \end{cases}$
Zastawny et al. [20]	$\begin{cases} C_D = C_{D,\alpha=90^\circ} + (C_{D,\alpha=90^\circ} - C_{D,\alpha=0^\circ})\sin^{a_0}\alpha \\ \text{where} \\ C_{D,\alpha=0^\circ} = \frac{a_1}{Re_p^{a_2}} - \frac{a_3}{Re_p^{a_4}} \\ C_{D,\alpha=90^\circ} = \frac{a_5}{Re_p^{a_6}} - \frac{a_7}{Re_p^{a_4}} \end{cases}$
	$C_L = \left(\frac{b_1}{Re_p^{b_2}} - \frac{b_3}{Re_p^{b_4}}\right)\sin(\alpha)^{b_5+b_6 Re_p^{b_7}}\cos(\alpha)^{b_8+b_9 Re_p^{b_{10}}}$
	$C_T = \left(\frac{c_1}{Re_p^{c_2}} - \frac{c_3}{Re_p^{c_4}}\right)\sin(\alpha)^{c_5+c_6 Re_p^{c_7}}\cos(\alpha)^{c_8+c_9 Re_p^{c_{10}}}$
Hoerner [23]	$\frac{C_L}{C_D} = \sin^2\alpha\cos\alpha$