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A full three dimensional numerical simulation of the sediment transport and the scouring at a rectangular obstacle

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ABSTRACT

We employ a numerical simulation of the three-dimensional fluid flow and the simultaneous transport of sediment to reproduce current-driven sediment transport processes. In particular, the scouring at a rectangular obstacle is investigated. To solve the instationary incompressible Navier–Stokes equations we use the code NaSt3D. The morphological change of the sediment bed is modeled by Exner's bed level equation, which is discretized and coupled to the discrete fluid model, i.e., to the NaSt3D code. A large eddy turbulence approach using a Smagorinsky subgrid scale tensor is applied. For our purposes, we only consider bed load transport under clear water conditions. Furthermore, we demonstrate mass conservation and convergence of our approach for a test case. We compare the results of our numerical simulations for a scour mark with those obtained in a laboratory flume. The typical sedimentary processes and the sedimentary form of a scour mark are well captured by our numerical simulation.

1. Introduction

Sediment transport processes and scouring effects are significant issues in hydraulic engineering. Usually, the physical processes of forming scour marks and sedimentary forms are studied in laboratory flumes. Such experiments are however time-intensive, costly and not always easy to conduct. Here, numerical simulation can help to reduce costs and to obtain a better understanding of the relevant flow and transport phenomena.

Fluvial obstacle marks are mainly generated by bed load transport, which is a driving constituent of sediment transport [41]. In case of a scour mark, sediment is entrained in front of an obstacle, the luff, and transported in the bed load layer around the obstacle. If the velocity then gets smaller than a critical value, sediment is deposited in the lee. The type of transport under clear water conditions is almost exclusively reptation. The involved processes and the resulting depositional bedforms are strictly three-dimensional. We present a numerical approach for their simulation and discuss the obtained results.

The remainder of this paper is organized as follows. In Section 2, we describe the fluid-sediment-model, which consists of the Navier–Stokes equations, the turbulence modeling approach, and Exner's bed level equation. In Section 3, we shortly discuss the numerical discretization and its properties. In Section 4, we compare the results of our numerical simulations to a scour mark studied in a laboratory flume. Some concluding remarks are given in Section 5.

2. Model: Navier–Stokes and Exner's bed level equation

Due to the complex three-dimensional character of the scour mark and other bedforms, it is necessary to use a full three-dimensional flow model. To this end, we use a single phase model. Here, for the flow problem, the instationary incompressible Navier–Stokes equations in their dimensionless form read as

$$\frac{\partial \mathbf{u}}{\partial t} + \nabla \cdot (\mathbf{u} \otimes \mathbf{u}) = \frac{1}{Fr}\mathbf{g} - \nabla p + \frac{1}{Re}\Delta \mathbf{u} \quad \text{in} \quad \Omega_f \in \mathbb{R}^3, \tag{1a}$$

$$\nabla \cdot \mathbf{u} = 0 \quad \text{in} \quad \Omega_f \in \mathbb{R}^3, \tag{1b}$$

where $\mathbf{u}$ denotes the velocity, p the pressure, and $\mathbf{g}$ the volume forces.

$$Re = \frac{\mathbf{u}_\infty \cdot d}{\nu} \tag{2}$$

denotes Reynolds number and

$$Fr = \frac{\mathbf{u}_\infty}{\sqrt{g \cdot d}} \tag{3}$$

the Froude number. Both numbers, Re as well as Fr, are dimensionless numbers which characterize the flow conditions. The characteristic

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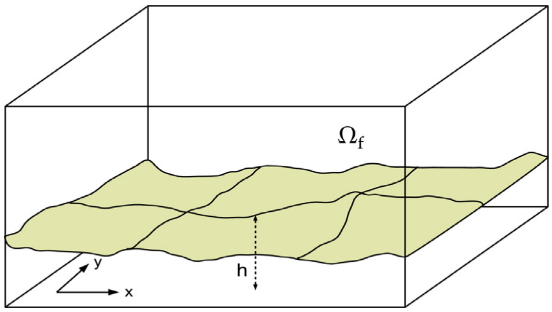

Fig. 1. The sediment surface is described by the height $h(x, y)$, i.e. the distance from a underlying plane (x, y). Slopes are denoted by ∇h and can be computed directly. Thus, the fluid domain Ω_f is bounded by $h(x, y)$ from below.

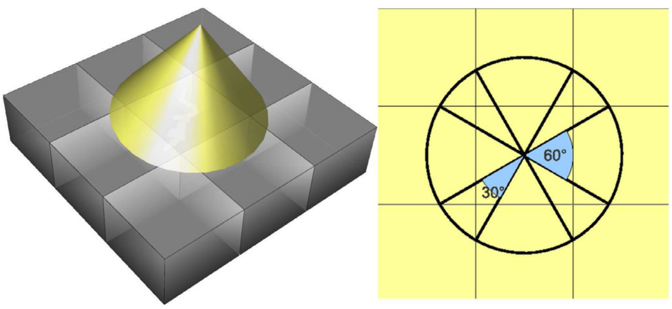

Fig. 2. If the slope angle surpasses the critical angle $\alpha > \alpha_c$, surplus masses are distributed to the adjacent cells in each iteration step. Lateral cells are corrected with $\frac{1}{12}$ of the surplus masses and diagonal cells with $\frac{1}{24}$, which includes a safety factor of $\frac{1}{2}$. This process is iterated over the whole sediment surface until the limit condition (13) is fulfilled for each cell in each direction.

length and velocity are denoted by d and $\mathbf{u}_\infty$. As commonly used, ν stands for the kinematic viscosity of the fluid.

To model the turbulence a Large Eddy Simulation (LES) is chosen. Details regarding turbulence models can be found in [31]. For the here described applications LES is viewed as the optimum for accuracy, computational efficiency and handling. An approach by Smagorinsky [38] is used as a sub-scale model. Applying a space averaging filter [35] to the Navier–Stokes Eqs. (1a) yields

$$\frac{\partial \overline{\mathbf{u}}}{\partial t} + \nabla \cdot (\overline{\mathbf{u}} \otimes \overline{\mathbf{u}}) = \frac{1}{Fr}\mathbf{g} - \nabla \overline{p} + \frac{1}{Re}\Delta \overline{\mathbf{u}} - \nabla \cdot \tau \tag{4}$$

where $\overline{u}$ and $\overline{p}$ are the filtered quantities. Eq. (4) now contains the additional sub-grid-scale tensor

$$\tau = -\nu_t \overline{D}_{ij} \tag{5}$$

Table 1
Selection of values for the critical angle of repose α_c [21,30]. The large variety and the measuring of the values under water allows only rough estimates. This fact has to be taken into consideration when validating the numerical experiments.

Sand	Dry to wet	$20--45^\circ$
Gravel	Roundness	$30--50^\circ$
Silt and clay	Shape and roughness	$20--60^\circ$

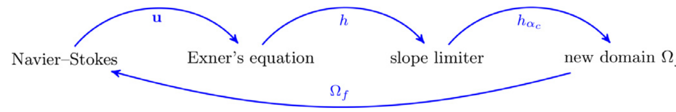

Fig. 4. Flow chart of our loosely partitioned coupling algorithm in each time step, the velocities from NaSt3D are used to calculate the new sediment height h, which, after correcting to h_{α_c} due to the slope limiter iteration, determines the new Ω_f and therefore the new fluid domain.

where the eddy-viscosity is denoted by $\nu_t = l^2|\overline{D}|$ with $\overline{D} = \sqrt{\frac{1}{2}\overline{D}_{ij}\overline{D}_{ij}}$ and

$$\overline{D}_{ij} = \left(\frac{\partial \overline{u}_i}{\partial x_j} + \frac{\partial \overline{u}_j}{\partial x_i}\right) \tag{6}$$

As the characteristic length l we use

$$l = C_s\sqrt{\Delta x^2 + \Delta y^2 + \Delta z^2}. \tag{7}$$

The Smagorinsky constant C_s is set to $C_s = 0.0825$.

The Navier–Stokes equations are solved on a fluid domain Ω_f. The bottom of this domain is bounded by the sediment surface $h(x, y)$. This sediment surface h describes the height of the underlying sediment with respect to a reference plane (x, y) further below, compare Fig. 1. To model the temporal change of the sediment surface h, we use the bed level equation postulated by [16], i.e.

$$\frac{\partial h}{\partial t} + \nabla \cdot \mathbf{q}_s(\tau(\mathbf{u})) = 0, \tag{8}$$

where $\mathbf{q}_s(\boldsymbol{\tau}(\mathbf{u}))$ is the transport rate function of the sediment. It depends on the shear stress τ, which is a function of the fluid velocity $\mathbf{u}$. Here, the shear stress function $\tau(\mathbf{u})$ is needed on the sediment surface. The Exner equation results from the conservation of mass and therefore from first principles. It states that the net balance between gain and loss of mass in a certain control volume results in a change of sediment height h. Several studies using the Exner equation to investigate the evolution of the geomorphology were conducted in e.g. [23,24,32,33]. Moreover, Coleman and Nikora [8] derived a version of

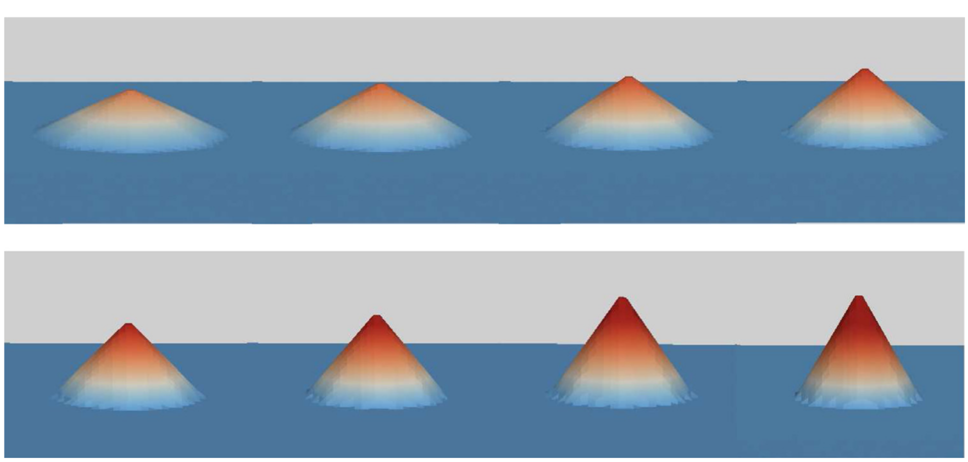

Fig. 3. Results of 8 test simulations with slope limiting algorithm given a sediment pile and critical angles from 25° to 60°. After 10 – –12 iterations all angle conditions are fulfilled while the masses are conserved.