Computers and Fluids 125 (2016) 59–70

Contents lists available at ScienceDirect

Computers and Fluids

journal homepage: www.elsevier.com/locate/compfluid

A shifting discontinuous-grid-block lattice Boltzmann method for moving boundary simulations

Nipun Arora [a], Amit Gupta [a,*], Wei Shyy [b]

[a] Department of Mechanical Engineering, Indian Institute of Technology, Hauz Khas, New Delhi, India
[b] Department of Mechanical and Aerospace Engineering, Hong Kong University of Science and Technology, Clear Water Bay, Hong Kong, China

ARTICLE INFO

Article history:
Received 20 July 2015
Revised 4 November 2015
Accepted 7 November 2015
Available online 17 November 2015

Keywords:
Lattice Boltzmann
Discontinuous-grid-block
Multi-relaxation time
Moving boundary

ABSTRACT

A translating discontinuous-grid-block model for moving boundaries of finite thickness based on multi-relaxation time version of lattice Boltzmann method has been developed. The implementation of this model to simulate moving boundary flows has been demonstrated for the cases of a cylinder in simple shear flow, a single rigid wing executing 'clap and fling' motion, and the propulsion of a plunging flat plate. A number of interpolation schemes of linear, quadratic and cubic natures are assessed around the discontinuous grid interface. It is shown that the implementation of a body-fitted refined mesh that moves along with the object reduces the spurious oscillations registered in the force and velocity measurements compared to a single coarse grid block. Moreover, use of multiple relaxation times helps overcome stability issues at high Reynolds number, normally encountered in the single-relaxation time model. Significantly, in the former model the same base grid could handle flows with good accuracy for $10 \leq Re \leq 1000$. The proposed technique offers significant advantage in terms of capturing flow around moving solids at lower computational cost and simulation time as compared to the stationary discontinuous-grid-block method.

1. Introduction

In the field of fluid mechanics, dealing with complicated processes like phase change and moving boundary flows has emerged as a big challenge, especially with ever increasing cost of experimental equipment and limited data availability [1]. However, with rapid improvements in computational facilities, design of systems via numerical analyses is being given more preference over conducting physical experiments. Most of these are based on the solution of Navier–Stokes equation and have been successful as well. However, the existing numerical schemes of re-meshing, grid generation and efficient matrix solvers, etc. that has garnered significant attention in the recent past suggest that there is still scope for improvement. Encountering various limitations in Navier–Stokes based mathematical models and experiencing complexities in implementation, researchers started opting for other alternatives. In the past decade, the lattice Boltzmann method (LBM) emerged as one such substitute which enjoys certain advantages over existing techniques. Some of the salient features are (a) the solution process is local in nature and hence there is no requirement of solving simultaneous linear algebraic equations which makes the solution process non-iterative and free of matrix inversions, and (b) it is easy to implement and parallelize.

By convention, LBM utilizes a regular, uniform and stationary Cartesian grid for solving the discretized Boltzmann equation where the particle distribution functions are calculated and whose hydrodynamic moments provide the macroscopic variables (density, velocity and temperature) [2–6]. The most commonly used version is the BGK (Bhatnagar, Gross and Krook) or also known as the single-relaxation time (SRT) model. Despite its advantages, the BGK model poses numerical instability at low values of relaxation time and hence is difficult to use for flows at higher Reynolds number (Re). Contrary to SRT, the multi-relaxation time (MRT) model exhibits better numerical stability even at high Re (i.e., at very low values of relaxation time) and has been quite successful in curtailing the spurious oscillations registered in force measurement [7,8].

Although quite successful in reducing noise and fluctuations observed in the calculation of forces, MRT does not eliminate them completely, especially for moving bodies. This is due to the variability in shape or volume of the object, i.e., the number of nodes or grid points inside the solid do not remain constant as these undergo transitions from non-fluid region to fluid region or vice versa due to the movement of the solid (which contributes to the noise). As demonstrated with the help of Fig. 1 which shows the standard LBM interpretation of a solid body immersed in fluid, this issue can be resolved by improving the resolution of moving body by refining mesh or grid

* Corresponding author. Tel.: +911126591270; fax: +91 11 26582053.
E-mail address: agupta@mech.iitd.ac.in, amitgupta2004@gmail.com (A. Gupta).

http://dx.doi.org/10.1016/j.compfluid.2015.11.003
0045-7930/

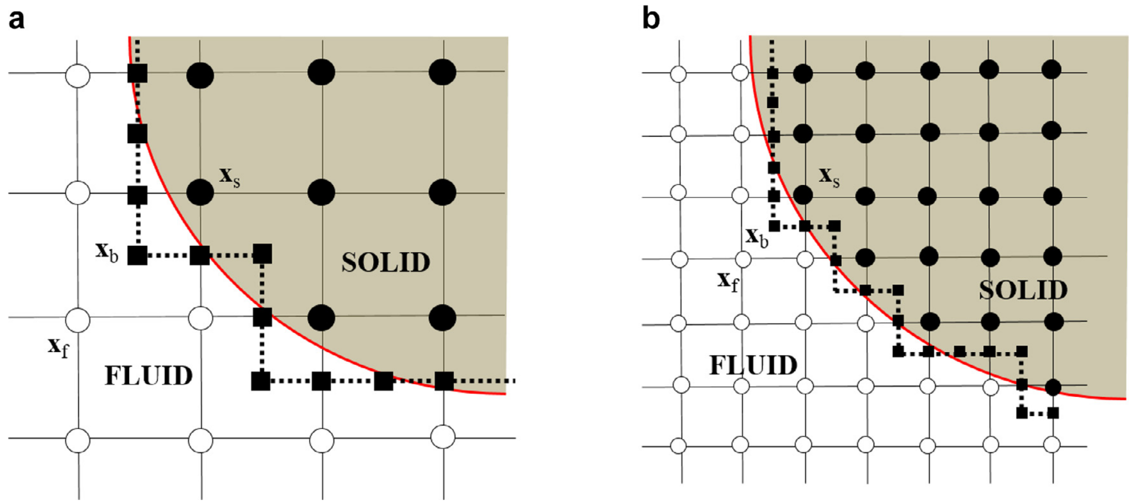

Fig. 1. Layout of the regularly spaced lattices and curved solid boundary (solid red). The hollow and filled circles denote fluid and solid nodes respectively. The solid squares denote the boundary nodes. The dotted line represents the halfway bounceback interpretation of the curved geometry with (a) normal and (b) increased resolution. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article).

size near the surface of the solid to capture the minutest of details. Here, the outline of a moving object is represented by a set of fixed nodes lying exactly halfway between the immediate solid and fluid nodes on either side of its boundary. The true shape of a moving body can also be preserved by the employment of the immersed boundary methods, where the boundary is represented by a set of nodes which are not stationary (as in standard LBM) but are constantly moving with the body [9]. But in either case, as the mesh size decreases, the resolution of an object will improve which can be expected to lead to increased accuracy in results obtained.

Regrettably, refinement of the grid throughout the domain (even in those regions where the flow is not expected to evolve rapidly and drastically) is computationally expensive and inefficient. This problem can be overcome by using a coarser mesh far away from the body thereby creating zones of different mesh sizes, which we term as the discontinuous-grid-block method in LBM. Creation of separate coarse and fine block can greatly reduce the computational time and memory in comparison to the employment of uniform grid throughout. In the past, several groups have demonstrated the principles of the discontinuous-grid-block method applied to flows past stationary solids [10–12]. However, for a moving body, developing a body-fitted mesh which is dynamic and moves along with the object on a ‘carpet’ of the coarse block is essential to get improved accuracy and at the same time maintaining a reasonable computational efficiency.

Thus, in the present study we extend the earlier work carried out by Yu et al. [10,11] and Peng et al. [12] on the stationary discontinuous-grid-block (SRT and MRT, respectively) by developing an algorithm for situations where the finer block is desired to translate with the moving solid (and in essence, a shifting discontinuous-grid-block LBM). Although the SRT version of the moving block LBM has been developed previously [13], the focus of this work is to highlight and illustrate the details of this translating multiblock method using MRT version of LBM. Additionally, the method as applied to the analysis of flows with $1 \le \mathrm{Re} \le 1000$ is also demonstrated and discussed from stability and accuracy considerations.

2. Methodology

2.1. Multi-relaxation time LBM

The detailed description of the SRT as well as MRT uniform grid models, calculation of macroscopic variables from the distributions, the force evaluation on a moving body using halfway bounceback method (as proposed by Ladd [6]) along with proper validations are provided in earlier publications [14,15]. However, since the multi-block method described here is based on the MRT version, it is reiterated here to avoid any confusion or discontinuity.

In this method, the distribution function f (as defined in the SRT model) in the discrete velocity space $\mathbf{B}$ is mapped onto the moment space $\mathbf{K}$ by using a transformation matrix M [8], i.e.,

$$\hat{f} = Mf \qquad and \qquad f = M^{-1}\hat{f} \tag{1}$$

where $\hat{f}$ is a column matrix consisting of moments of the velocity distribution function (each row vector) which represents the following quantities in two-dimensional space:

$$\hat{f} = [\rho, e, \varepsilon, j_x, q_x, j_y, q_y, p_{xx}, p_{xy}]^T \tag{2}$$

where ρ is the density, e signifies the kinetic energy, ε epitomizes the square of kinetic energy, j_x and j_y are the x and y components of the momentum flux, q_x and q_y are related to the x and y components of energy density, and p_{xx} and p_{xy} refer to the diagonal and off-diagonal terms in viscous stress tensor. Since the moments represent different physical quantities, MRT enjoys an added advantage of independently varying the relaxation time scales. Also, the transformation matrix is orthogonal (i.e. $M.M^T = I$) which ensures that the relaxation time matrix in moment space $\mathbf{K}$ comes out to be diagonal [16].

The equilibrium values for the non-conserved moments can be obtained from the conserved moments ρ and $\boldsymbol{j}$ (j_x and j_y) [8] as

$$e^{eq} = -2\rho + \frac{3}{\rho}\left(j_x^2 + j_y^2\right) \tag{3}$$

$$\varepsilon^{eq} = \rho - \frac{3}{\rho}\left(j_x^2 + j_y^2\right) \tag{4}$$

$$q_x^{eq} = -j_x, \quad q_y^{eq} = -j_y \tag{5}$$

$$p_{xx}^{eq} = \frac{1}{\rho}\left(j_x^2 - j_y^2\right) \tag{6}$$

$$p_{xy}^{eq} = \frac{1}{\rho} j_x j_y \tag{7}$$

The transformation matrix (orthogonalized by Gram-Schmidt procedure [17]) as employed by Lallemand and Luo [8] for the D2Q9