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Hybrid prediction of the aerodynamic noise radiated by a rectangular cylinder at incidence

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ABSTRACT

The acoustic radiation by a laminar flow over a rectangular cylinder at incidence is predicted using a two-step approach. The acoustic pressure is evaluated from the compact source approximation of Curle's analogy, where the fluctuation of the aerodynamic force is the source quantity. The latter is provided by numerical simulation of the incompressible flow, the presence of the bluff body being modelled via an immersed boundary method. The approach is validated by comparison with a direct noise computation of the aeolian tone produced by the flow over a circular cylinder at $Re = 150$ and $M = 0.2$. Ten values of incidence are considered, from 0° to 90° for the 2D flow, at $Re = 200$, over the rectangular cylinder, whose aspect ratio is 4. The acoustic power is strongly enhanced in comparison with the circular cylinder (by 6–15 dB) and with the case without incidence (by 30–40 dB). The contribution of the drag dipole is also significantly increased. The relative fluctuations of lift and drag drive the directivity for each case. Depending on the incidence, a block rotation of $\pm 15^\circ$ is observed on the directivity diagram. This is closely linked to the wake organisation, in particular the position of the stagnation point, and the orientation of the fluctuation of the aerodynamic force, all of these features undergoing a qualitative change at an incidence angle of 40°. One of the key results is that the acoustic efficiency increases quadratically with respect to the Mach number and to (*rms*) fluctuations of lift and drag coefficients, and depends linearly on the Strouhal number.

1. Introduction

The problem addressed here is that of airframe noise prediction, in order to investigate the effect of geometrical changes on acoustic emission. For relatively complex shapes, aeroacoustic design can be aided by parametric studies. Significant computational resources are necessary to generate the source fields associated with each shape; theoretical models may not account for all geometrical details. Typical examples of such configurations are landing gear, which is the dominant source at landing, and car side mirrors, whose tones lead to annoyance in the passenger compartment.

For the estimate of the noise radiated by an unsteady flow over stationary, rigid bodies, Curle's development [1] of Lighthill's analogy [2] is the most popular and practical formula. It yields a scaling of the acoustic intensity with the sixth power of the Mach number in the case of a compact body. This is due to the dipolar nature of the wall pressure term, which thus dominates the Lighthill quadrupole contribution at low Mach number. However, in the presence of the dipole, the quadrupole yields a component of the acoustic intensity that scales with the seventh power of the Mach number, as recalled by Spalart [3]. The Mach number up to which this hierarchy is maintained has not yet been clearly identified.

With the development of computational resources and architectures, the direct computation of aerodynamic noise (DNC) by solving the compressible Navier–Stokes equations becomes feasible at higher and higher Reynolds numbers, for a limited number of relatively simple shapes: circular and square cylinders, airfoils, sets of these (rod-airfoil configurations, cylinder tandems, side-by-side arrangement). Inoue & Hatakeyama [4] performed DNC for the 2D circular cylinder at $Re = 150$ at three subsonic Mach numbers, providing a reference solution for the validation of hybrid methods. They also illustrated the dipolar characteristic of the acoustic field, predicted by Curle's theory and generated by lift fluctuations that dominate the drag fluctuations at half their frequency. Increasing the Mach number allowed the role of the Doppler effect in the orientation of the wave-fronts [5] to be established.

Though parametric studies of shape and regime may be more easily addressed experimentally [6,7], numerical simulations

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provide full flow information that is useful for the analysis and modelling of source mechanisms in flows over bluff bodies, as exemplified by the three following contributions. Firstly, using a tailored Green function, Gloerfelt et al. [8] illustrated, numerically, the theoretical result according to which the surface term (dipole) in Curle's formulation is equivalent to the diffracted part of the pressure field associated with the volume term (quadrupole). Secondly, Curle's power law is based on a reference length, whose most natural choice in the case of the aeolian tone is the cylinder diameter, while the relevant choice in other cases has yet to be established. The blockage length (projected length in the transverse direction) may not be relevant, as shown for instance by Inasawa et al. [9], who obtained by DNC different noise levels while varying the streamwise length of rectangles without incidence and keeping the blockage ratio constant. They also show that a short streamwise length reinforces the drag dipole via a closer vortex generation, and exhibits a monopole source, weaker than, but with a stronger Mach number scaling than, and out of phase with, the drag dipole. Finally, Wolf et al. [10] carried out a parametric study of the wake interaction between an airfoil and a relatively small circular cylinder in its vicinity. For two positions of the cylinder and three subsonic Mach numbers, their numerical results emphasise an intense dipolar interference at the frequency of the cylinder vortex shedding.

In that context, the availability of noise prediction methods, flexible with respect to the body shape, is a crucial issue in view of the analysis and modelling of airframe noise, where the effects of geometry are of interest. The goal of the present effort is to prefigure a numerical aeroacoustic facility for relatively complex geometries, so that a set of shapes can be tested, allowing us to search for relevant quantities (e. g. characteristic lengths) to use in scaling laws. A hybrid method is thus proposed, based on the coupling of Curle's analogy with an immersed boundary method (IBM), which ensures flexibility with respect to the body shape. As an application of the method, the noise radiated by the flow over a rectangular cylinder at Re = 200 is computed, the incidence being varied while the blockage length is maintained constant. The latter turns out to be irrelevant in the scaling law, consistent with the results of Inasawa et al. [9]. The correlation of the acoustic field with the flow statistics amounts to a scaling of the acoustic power with the fluctuating aerodynamic force, which comes as a generalisation of Phillips' formula [11] for aeolian tones.

The paper is organised as follows. In Section 2, the hybrid tool is presented and validated. The approximation of Curle's formula for compact bodies is recalled before estimate of the aerodynamic force using IBM is detailed. The application to the flow over a rectangular cylinder in ten cases of incidence is discussed in Section 3, where the unsteady aerodynamics are analysed and correlated to the acoustic directivity and power. The scaling law is derived in Section 4 while the results and the limitations of the hybrid method are further discussed in Section 5.

2. Coupling Curle's analogy with immersed boundary method

As explained in the introduction, flexibility with respect to the body shape is targeted along the prediction process of both the acoustic field and the unsteady flow. In the present study, such numerical aeroacoustic facility consists of Curle's integral solution, fed by the unsteady aerodynamic force that is provided by a prior flow simulation using an Immersed Boundary Method (IBM) to model the no-slip condition at the body wall. Those two elements and their coupling are described in the two following subsections for a two-dimensional model, then the prediction of the aeolian tone is conducted for validation concern.

2.1. Noise estimate method

The acoustic part of the hybrid method computes the convected form of Curle's integral solution [1] to Lightill's equation, in the frequency domain. According to this, the acoustic pressure for an observer located in $\mathbf{x} = (x_1, x_2)$ is given by:

$$\tilde{p_a}(\mathbf{x},\omega) = -\oint_{\Sigma}\left[\tilde{p}\delta_{ij} - \tilde{\tau}_{ij}\right]\mathbf{n}_j \frac{\partial \tilde{G}_c(\mathbf{x}|\mathbf{y},\omega)}{\partial y_i}\mathrm{d}\sigma(\mathbf{y}) \quad (1)$$

where, as also sketched in Fig. 1a, Σ is the body surface and $\mathbf{n}$ its outward normal, $\mathrm{d}\sigma(\mathbf{y})$ is the elementary surface, $\tilde{f}$ is the Fourier transform of f, p is the pressure, $\tau_{ij} = \frac{1}{\mathrm{Re}}\frac{\partial u_i}{\partial y_j}$ are the viscous stress tensor components, δ_{ij} is Kronecker's symbol, and ω is the angular frequency under consideration.

The 2D free-field convected Green function $\tilde{G}_c$ is given in the frequency domain by [12–14]

$$\tilde{G}_c(\mathbf{x}|\mathbf{y},\omega) = \frac{i}{4\beta}\exp\left(\frac{i\mathrm{M}kr_1}{\beta^2}\right)H_0^{(2)}\left(\frac{kr_\beta}{\beta^2}\right) \quad (2)$$

where $\mathbf{y} = (y_1, y_2)$ is the source position, $r_i = x_i - y_i, i^2 = -1, H_\nu^{(m)}$ is the Hankel function of order ν and kind $m, k = \omega/c_0$ and c_0 is the sound speed in the uniform medium at rest. Moreover, $\beta^2 = 1 - \mathrm{M}^2$, where M is the Mach number of the flow in the observer domain, is the Prandtl-Glauert factor, and $r_\beta = \sqrt{(x_1-y_1)^2 + \beta^2(x_2-y_2)^2}$. The first space derivatives of that Green function are [13]:

$$\frac{\partial \tilde{G}_c}{\partial y_1} = K\frac{-ik}{4\beta^3}\left[i\mathrm{M}H_0^{(2)}\left(\frac{kr_\beta}{\beta^2}\right) - \frac{r_1}{r_\beta}H_1^{(2)}\left(\frac{kr_\beta}{\beta^2}\right)\right]$$
$$\frac{\partial \tilde{G}_c}{\partial y_2} = K\frac{i}{4\beta}\frac{kr_2}{r_\beta}H_1^{(2)}\left(\frac{kr_\beta}{\beta^2}\right) \quad (3)$$

with $K = \exp\left(\frac{i\mathrm{M}kr_1}{\beta^2}\right)$. In the present application of Curle's analogy, which is devoted to low speed flows, the volume source terms have been neglected to get (1). Their quadrupolar character may *a priori* make them insignificant with respect to the surface terms for low Mach number subsonic aeroacoustics. However, such hierarchy between multipole sources relies on the source compactness that assumes there is no delay between emission times from different source points [3].

Assuming a compact source and a far field estimate, $||\mathbf{x} - \mathbf{y}||$ can be approximated by $||\mathbf{x}||$, that is $r_i \approx x_i$ and $r_\beta \approx \sqrt{x_1^2 + \beta^2 x_2^2}$. Consequently, the Green function and its derivatives do not depend on $\mathbf{y}$ anymore, and Curle's solution reduces to:

$$\tilde{p_a}(\mathbf{x},\omega) = \partial\check{G}_{c,i}(\mathbf{x},\omega)\tilde{F}_i(\omega) \quad (4)$$

where $\partial\check{G}_{c,i}$ stands for the approximation of $\partial\tilde{G}_c/\partial y_i$ when $||\mathbf{y}|| \ll ||\mathbf{x}||$, and

$$\tilde{F}_i(\omega) = -\oint_{\Sigma}\left[\tilde{p}\delta_{ij} - \tilde{\tau}_{ij}\right]\mathbf{n}_j\mathrm{d}\sigma(\mathbf{y}) \quad (5)$$

is the ith component of the unsteady aerodynamic force on the bluff body, here including its viscous part. In A, the approximation of the derivatives of $\tilde{G}_c$ by $\mathrm{d}\check{G}_c^{(i)}$ is tested, the error being characterised as a function of the acoustic wavenumber and the observer distance, for different Mach numbers. The results show that when the acoustic wavelength is greater than fifty source lengths and the observer distance is greater than four source lengths, the error stays under one percent up to $\mathrm{M} = 0.5$. It thus appears that the geometric far-field is a less restricting assumption than the acoustic compactness, which limits the validity for high frequencies.

Eq. (4) is of great practical interest, for it yields the acoustics directly from the aerodynamic force, which is thus the only source quantity to be stored. However, its full time series (or frequency