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Analytical solutions for hydromagnetic natural convection flow of a particulate suspension through isoflux–isothermal channels in the presence of a heat source or sink

Ali J. Chamkha *, Seham S. Al-Rashidi

Manufacturing Engineering Department, The Public Authority for Applied Education and Training, Shuweikh 70654, Kuwait

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ABSTRACT

This work considers the problem of steady natural convection hydromagnetic flow of a particulate suspension through an infinitely long channel in the presence of heat generation or absorption effects. The channel walls are maintained at isoflux–isothermal condition. That is, the thermal boundary conditions are such that one of the channel walls is maintained at constant heat flux while the other is maintained at a constant temperature. Various closed-form solutions of the governing equations for different special cases are obtained. A parametric study of the physical parameters involved in the problem is done to illustrate the influence of these parameters on the velocity and temperature profiles of both phases.

1. Introduction

Natural convection flow of a two-phase (fluid/particle) suspension represents one of the most interesting and challenging areas of research in heat transfer. Such flows are found in a wide range of applications including processes in the chemical and food industries, solar collectors where a particulate suspension is used to enhance absorption of radiation, cooling of electronic equipments, and cooling of nuclear reactors. Very little work has been done on natural convection for a two-phase particulate suspension. Most work on natural convection flows within vertical parallel-plate channels are done only for a single phase. The evolution of cooling technology includes the progressive research of using natural convection, which is an inexpensive mode of heat transfer in electronic equipments cooling. Vertical plates and channels are of the most encountered configurations used in natural convection cooling of electronic equipment.

A literature review in general for the historical papers reported in the development of cooling technology for electronic equipments has been presented by Bergles [6]. Later, an extensive review of electronic equipment cooling by different modes of heat transfer has been presented by Incorpera [9]. The review includes natural convection heat transfer in parallel channels, inclined channels and enclosures as well as other configurations with different operating conditions. The importance of heat transfer considerations in the design of electronic equipment has been studied extensively and reported by Aung and Chaimah [4], Jaluria [10], Kraus and Bar-Cohen [12], and Steinberg [17].

Akbari and Borgers [1] studied free convection laminar heat transfer between the channel surfaces of the Trombe wall. The study was done using a line-by-line forward marching implicit finite-difference technique. The study was restricted to laminar flow between two parallel plates, each at some effective uniform temperature. Yao [18] investigated the problem of mixed convection in vertical channel. An analytical solution is developed to study the hydrodynamically and thermally developing laminar flow in a heated channel. The transient effects in natural convection cooling of vertical parallel plates are reported by Joshi [11]. Aung [3] considered fully developed laminar free convection between vertical plates heated asymmetrically. Aung et al. [5] reported on the development of laminar free convection between vertical flat plates with asymmetric heating. A more detailed reference list was given by Muhanna [14] who investigated numerically laminar natural convection flows in obstructed vertical channels. Related references for natural and mixed convection flows of a single phase are given in the book by Gebhart et al. [8].

On the other hand, very little work has been reported on natural convection flow of a particle–fluid suspension over and through different geometries. Chamkha and Ramadan [7] and Ramadan and Chamkha [16] have reported some analytical and numerical results for natural convection flow of a two-phase particulate suspension over an infinite vertical plate. Also, Okada and Suzuki [15] have considered buoyancy-induced flow of a two-phase

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* Corresponding author.
E-mail address: achamkha@yahoo.com (A.J. Chamkha).

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Nomenclature

$\vec{B}$	magnetic induction
c	fluid-phase specific heat at constant pressure
c_p	particle-phase specific heat at constant pressure
g	gravitational acceleration
Gr	Grashof number
h	channel width
H	dimensionless buoyancy parameter
k	fluid-phase thermal conductivity
M	Hartmann number
N	interphase momentum transfer coefficient
N_T	interphase heat transfer coefficient
P	fluid-phase hydrostatic pressure
Pr	fluid-phase Prandtl number
Q	heat generation/absorption coefficient
q_1	wall heat flux
r_{qt}	walls thermal ratio
T	fluid-phase temperature
T_p	particle-phase temperature

u	fluid-phase dimensionless velocity
u_p	particle-phase dimensionless velocity
U	fluid-phase velocity
U_p	particle-phase velocity
x, y	cartesian coordinates

Greek symbols

α	velocity inverse Stokes number
γ	specific heat ratio
ε	temperature inverse Stokes number
η	dimensionless y-coordinate
θ	dimensionless fluid-phase temperature
κ	particle loading
μ	fluid-phase dynamic viscosity
ρ	fluid-phase density
ρ_p	particle-phase density
σ	fluid-phase electrical conductivity
ϕ	dimensionless heat generation/absorption coefficient

suspension in an enclosure. Al-Subaie and Chamkha [2] performed an analytical study dealing with natural convection flow of a particulate suspension through a vertical channel with isothermal walls. However, the present authors were unable to locate any theoretical or experimental work in the literature dealing with natural convection laminar flow of a particulate suspension in isoflux–isothermal vertical channels. This is the objective of the present work. In the formulation of the general problem, magnetic effects which affect the flow if the fluid is electrically conducting and heat generation or absorption effects which are important in situations where a heat source or sink may be placed within the flow are included.

2. Problem formulation

Consider steady, laminar, natural convection flow of a particulate suspension in a vertical parallel-plate channel. The channel walls are maintained at the isoflux–isothermal condition. The schematic of the problem is shown in Fig. 1. The fluid phase is assumed to be Newtonian, viscous, electrically conducting, and heat generating or absorbing. The particle phase is assumed to be made up of discrete particles of one size and constant density. The particle phase is assumed to be pressure-less and electrically non-conducting. Both phases are assumed to be interacting continua. The governing equations for this investigation are based on the balance laws of mass, linear momentum and energy for both the fluid and particle phases. For small volume fraction of particles [13], they can be written in vector form as

$$\vec{\nabla}\cdot\left(\rho\vec{V}\right)=0 \tag{1}$$

$$\rho\vec{V}\cdot\vec{\nabla}\vec{V}=-\vec{\nabla}P+\vec{\nabla}\cdot\left(\mu\vec{\nabla}\vec{V}\right)-\rho_pN\left(\vec{V}_p-\vec{V}\right)+\rho\vec{g}+\sigma(\vec{V}\times\vec{B})\times\vec{B} \tag{2}$$

$$\rho c\vec{V}\cdot\vec{\nabla}T=\vec{\nabla}\cdot\left(k\vec{\nabla}T\right)+\rho_pc_pN_T(T_p-T)\pm Q(T-T_o) \tag{3}$$

$$\vec{\nabla}\cdot\left(\rho_pV_p\right)=0 \tag{4}$$

$$\rho_p\vec{V}_p\cdot\vec{\nabla}\vec{V}_p=\rho_pN(\vec{V}_p-\vec{V})+\rho_p\vec{g} \tag{5}$$

$$\rho_pc_p\vec{V}\cdot\vec{\nabla}T_p=-\rho_pc_pN_T(T_p-T) \tag{6}$$

where $\vec{V}$ and $\vec{V}_p$ are the velocity vectors of the fluid and particle phases, respectively. T and T_p are the temperatures of the fluid and particle phases, respectively. $\vec{g}$ is the gravity vector, $\vec{\nabla}P$ is the pressure gradient vector, T_o is the temperature at a reference point "o" in the channel, $\vec{B}$ is the magnetic induction vector and Q is the heat generation or absorption coefficient depending on its sign. The other parameters, namely, ρ, μ, σ, c and k are the density, dynamic viscosity, electrical conductivity, specific heat and thermal conductivity of the fluid phase, while, ρ_p and c_p are the particle-

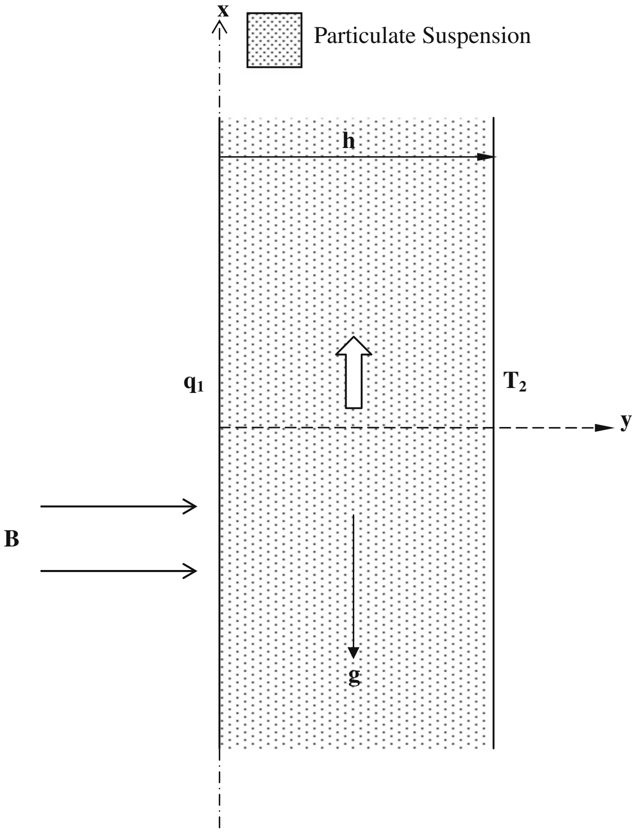

Fig. 1. Schematic of the problem.